MAXWELL WHEEL

Objective

To test the conservation of energy in a system with gravitational, translational and rotational energies.

Introduction

A wheel is suspended by two cords wrapped on its axis. After being released from rest, it unrolls from its cords and moves downward in the gravitational field. Figure 1 shows the initial stage. As time increases, more and more of the initial gravitational potential energy U_g of the system is converted to translational kinetic energy K_T and rotational kinetic energy K_R.

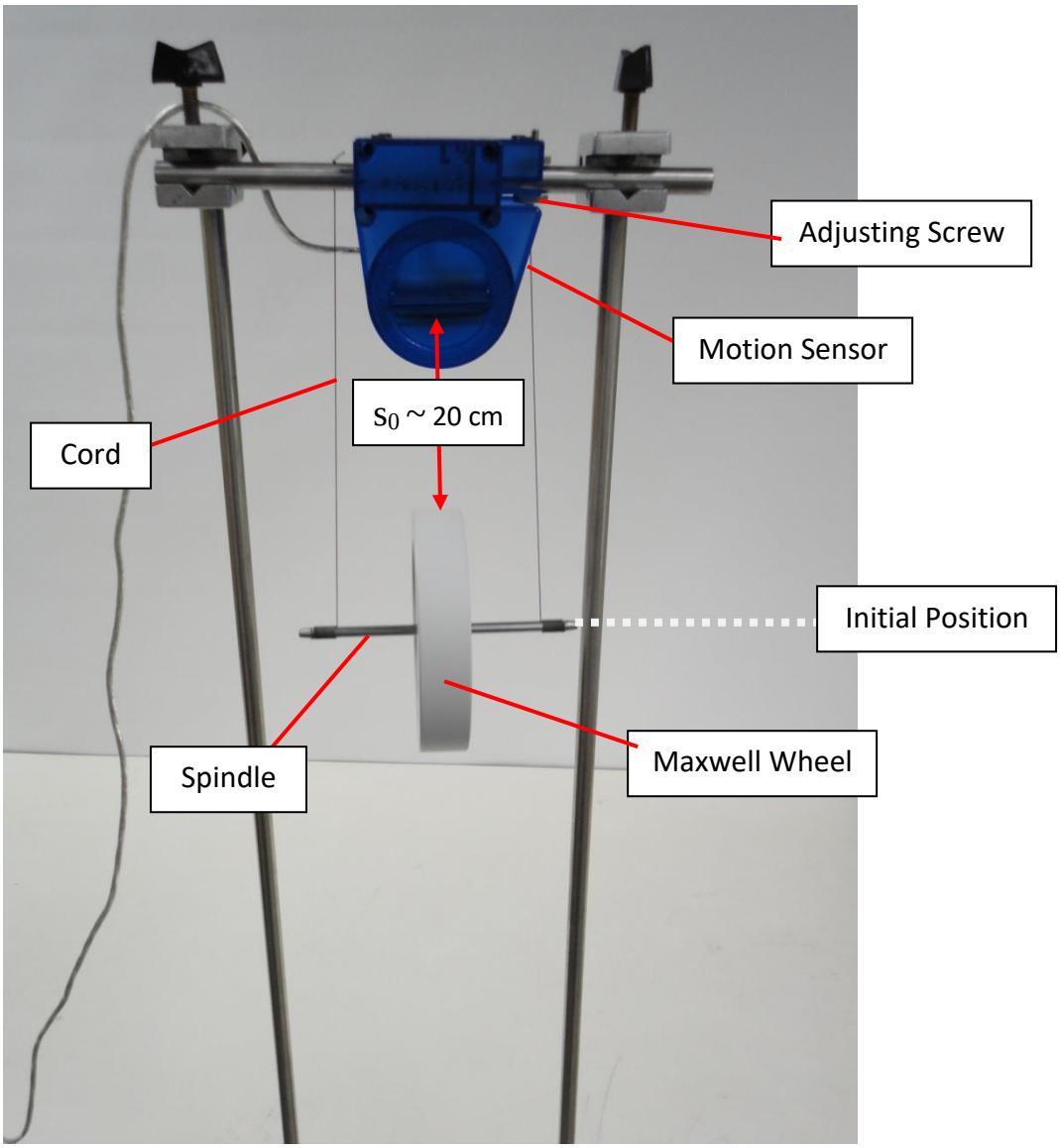


Figure 1

The total energy $E = U_g + K_T + K_R$ of the system is conserved if friction is negligible.

This implies that $E = E_{initial}$

That is, $U_g + K_T + K_R = U_{gi} + K_{Ti} + K_{Ri}$

If the system starts from rest, then its initial linear velocity v_0 and initial angular velocity ω_0 are zero. This implies that

$$K_{Ti} = \tfrac{1}{2}\, m\, v_0^2 = 0$$

$$K_{Ri} = \tfrac{1}{2}\, I\, \omega_0^2 = 0$$

where, m is the mass of the Maxwell wheel-spindle system and I is its moment of inertia about the axis of rotation.

Therefore, $U_g + K_T + K_R = U_{gi}$

This means that if there are no friction losses, the energy of the system at any point will remain constant and be equal to its potential energy at the start.

Now, we want to express $K_T = \tfrac{1}{2}\, m\, v^2$ and $K_R = \tfrac{1}{2}\, I\, \omega^2$ in terms of variables that can be easily measured in this lab.

We will release the wheel with zero initial velocity, and the center of mass of the wheel goes down with constant linear acceleration a.

Therefore, from $v^2 = v_0^2 + 2\, a\, y$

$$v^2 = 2\, a\, y$$

Furthermore, if the radius of spindle of the wheel is r, see Figure 2, then

$$\omega = \frac{\mathrm{v}}{r}$$

Therefore,

$$K_T = \frac{1}{2} m \mathrm{v}^2 = m a y$$

$$K_R = \frac{1}{2} I \omega^2 = \frac{I}{r^2} a y$$

Note that as y increases, both kinetic energies increase.

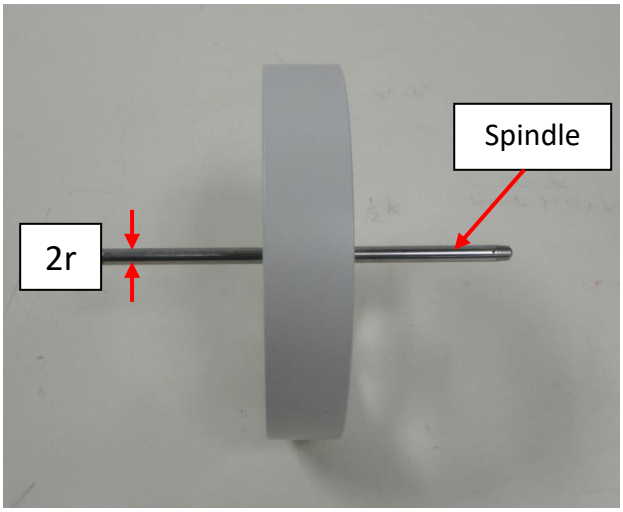


Figure 2

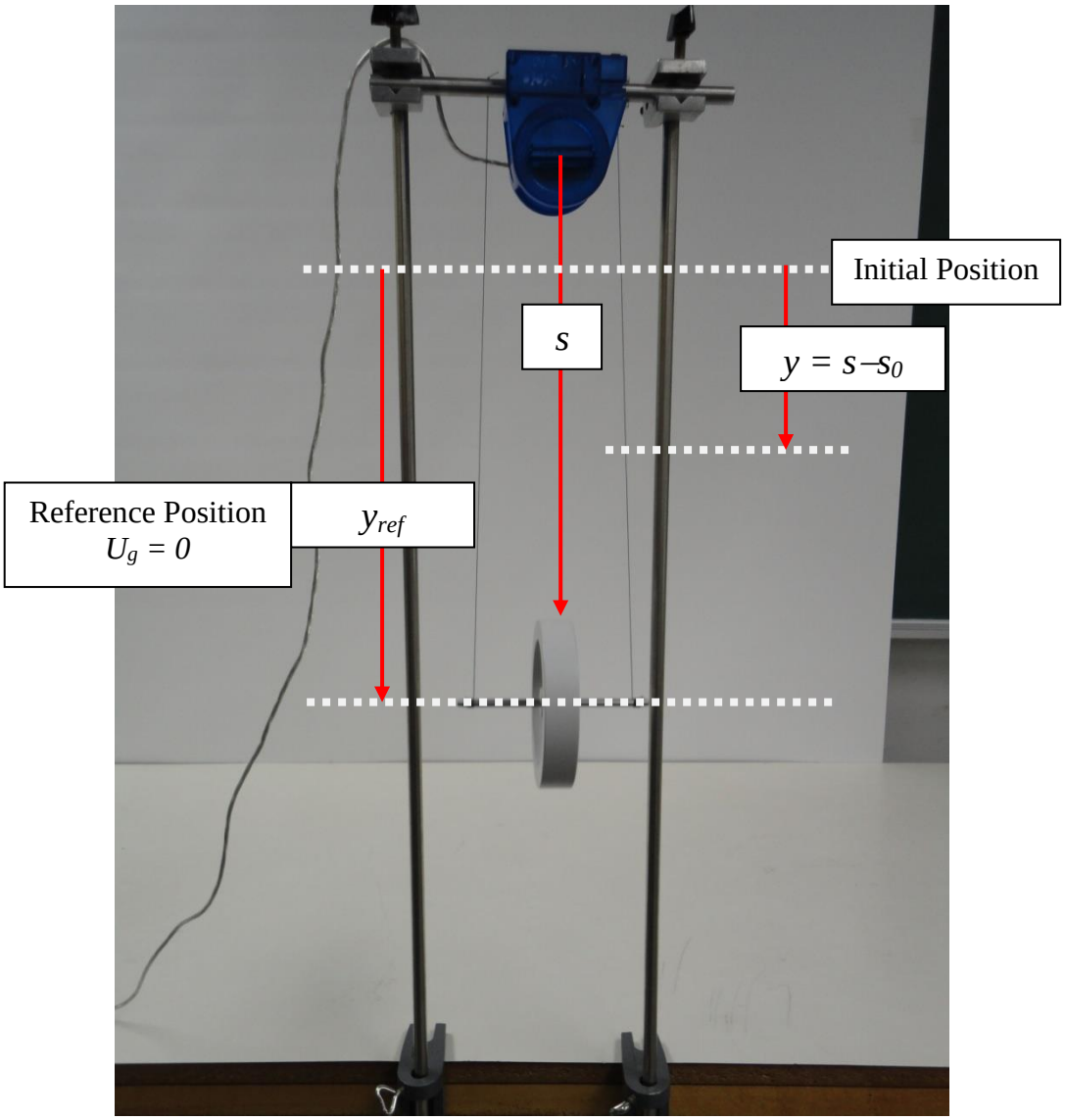


Figure 3

The gravitational potential energy of the system can be expressed as

$$U_g = m\,g\,(y_{ref} - y)$$

where, y is the vertical distance traveled, see Figure 3, and we have set the value of the gravitational potential energy to zero at $y = y_{ref}$. That is,

$$U_g = 0 \text{ at } y = y_{ref}.$$

The motion sensor measures the distance away from it as positive, and since the sensor is mounted above the moving object the downward direction will be taken as positive by the sensor. **Therefore, $U_g = mg(y_{ref} - y)$, and not $mg(y - y_{ref})$.** Note that as y increases, the potential energy decreases.

In this lab, you will use a motion sensor with DataStudio for collecting data and Excel for analyzing the data. You will verify that the linear acceleration of the system is constant. You will notice that r being very small compared to the radius of the wheel, $\frac{I}{r^2}$ is much greater than m and thus K_T is very small compare to K_R. You will also verify that all three energy terms are linearly proportional to the vertical displacement y and the potential energy reference is entirely arbitrary. That is, only the changes in potential energy are physically meaningful, and hence you can add a constant to the potential energy without affecting the physics.

Part 1 – Finding the constant acceleration *a*

In this exercise, you will determine the acceleration by fitting position versus time graph to a quadratic function, $Y = A\,t^2 + B\,t + C$. The second derivative of the function is the acceleration, which is equal to $2A$.

1. Download the file **Maxwell Wheel.ds** from the link in phys101 homepage and save it on the desktop. This file has been preconfigured for optimum experimental parameters such as position and time scales.

2. From the **Start** batton, go to **All Program → DataStudio → English**. This will open DataStudio. Click on ***Open Activity***, navigate to the folder desktop and to the file **Maxwell Wheel.ds**, and open it. This will open the position-time graph.

3. Make sure the motion sensor is connected to a computer through the USB interface, and its range switch is set to work with the (short range) option. Also make sure the *Ultrasonic Transducer* is horizontal. You can use the *Rotating Head* to adjust the angle of the sensor, if needed. This alignment is very important for getting good results.

4. Make sure the wheel is hanging horizontally. If not, use the adjusting screw to make it horizontal. Carefully roll up the wheel on its cords until it is about 20 cm from the sensor. This is important because the motion sensor cannot detect any object within 15 cm. Be sure that the cord winding runs inward (see Figure 1), so that the wheel will descend smoothly. If at any time the wheel begins to wobble and move erratically, you must gently catch it and do a proper job of rewinding.

5. Release the system from rest and click the ***start*** button in DataStudio simultaneously. This step should preferably be done by the same person. Click the ***stop*** button just before the system unwinds completely.

6. Your data will look like the red curve in Figure 4. Click on the *Fit* button in DataStudio and then select *Quadratic Fit*. The blue curve in Figure 5 shows the fit. The fitting parameter A is related to the acceleration a of the system, through $a = 2A$. Note that a perfect quadratic fit would confirm the constant acceleration. Make sure the initial slope (parameter B) is zero (or very close to zero), which confirms $v_0 = 0$. If not, repeat the experiment. The magnitude of the value of B from the quadratic fit should be < 0.01 to ensure initial speed is very close to zero.

7. Write down the acceleration in your report.

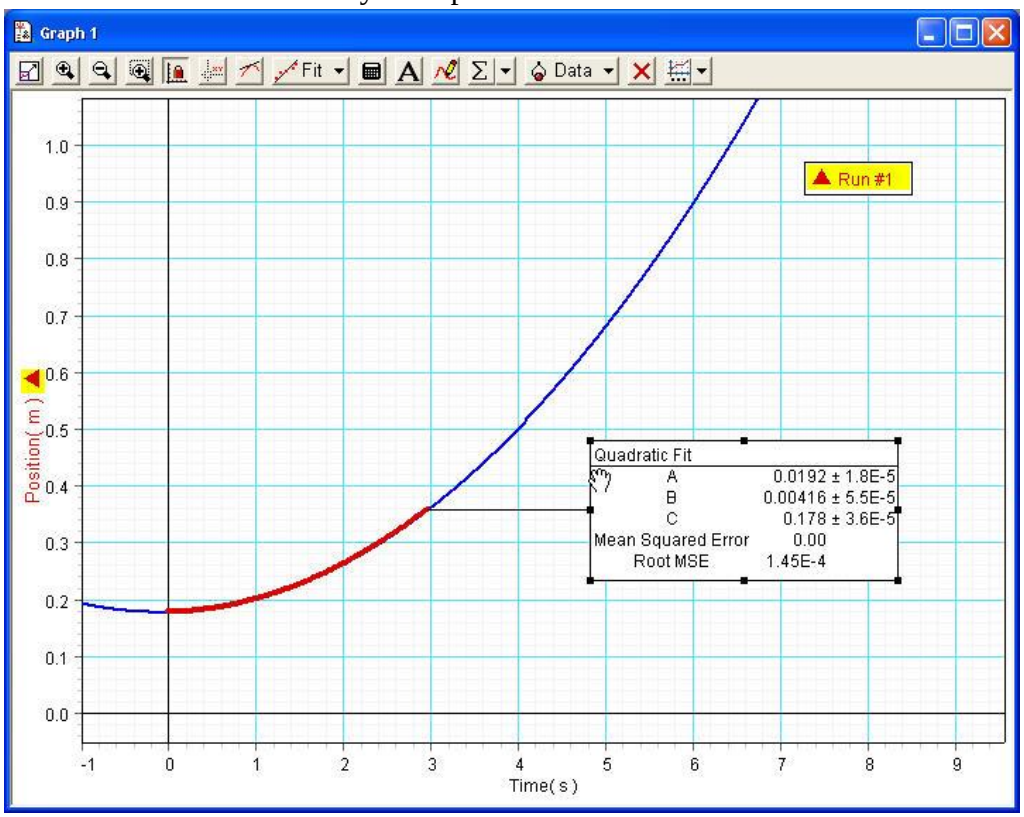


Figure 4

Part 2 – Testing conservation of energy

In this exercise, you will test the conservation of energy by plotting K_T, K_R, U_g and E versus y.

1. The diameter of the spindle is measured to be 6.0 mm. record this value in your report.

2. Export your data of Exercise 1 from DataStudio to Excel. To do this, choose ***Export Data…,*** from ***File*** menu. You will be presented with the ***Export Data*** box. Make sure you have chosen ***position*** data from the list and then click ***OK*** button. Save the data as text file in the desktop with an appropriate name (e.g. position.txt)

3. Now open Microsoft Excel. Click the ***office*** button  and click ***Open***. Navigate to the folder where you have saved the *position.txt* data file and open that file. (If you don't see the file, choose ***All files(*.*)*** in the ***files of type***). This will bring up the ***text import wizard*** – *step 1 of 3*. Just click the ***Finish*** button.

4. The first column of the data is time in seconds and the second column is the position (s) of the wheel, in meters, relative to the sensor. To find y, the vertical distance travelled, the initial-position value s_0 should be subtracted from the position values. Do this in column C. Follow the instructions from Step 1 to Step 2 in the Appendix at the end of this write up and then return here to Step 5.

5. Now you have to input the values of radius of the spindle r, reference point for potential energy y_{ref}, mass m, moment of inertia I, acceleration a and define names for them. Choose the y_{ref} as the final y-value of your data. Follow the instructions from Step 4 to Step 7 in the Appendix and then return here to Step 6.

6. Calculate in Excel K_T, K_R, U_g, and E using the equations given in the Introduction section. Record these values in columns D, E, F, and G respectively. To do this, follow the instructions from Step 9 to Step 10 in the Appendix and then return here to Step 7.

7. To plot K_T, K_R, U_g, and E as a function of y on the same graph, select the values, including the labels, in columns C, D, E, F, and G, and insert scatter plot. Answer the following depending on the plot generated using Excel:
   a) Describe the behavior of the translational kinetic energy K_T.
   b) Describe the behavior of the rotational kinetic energy K_R .
   c) Describe the behavior of the gravitational potential energy U_g .
   d) Describe the behavior of the total mechanical energy E.

8. In fact, E decreases slightly as y increases, suggesting the presence of non-conservative force such as rolling friction between the cord and spindle. Since the change in mechanical energy is the work done by the non-conservative forces on the system, the slope of the E versus y graph represents the rolling friction. Find the linear trendline of the E versus y graph. The absolute value of the slope is the magnitude of friction. Note that it is very small. Write down the magnitude of the force of friction.

Part 3 – Reference for gravitational potential energy is arbitrary

In this exercise, you will confirm that physics does not change by changing the reference for gravitational potential energy.

1. In your Excel worksheet of Exercise 2, choose the y_{ref} as the initial y-value (zero) of your data, type 0 in cell **J3**, and press *Enter* key . This will automatically change U_g values in column F and re-plot the graph. Note that although the appearance of the graph has changed, the physics is not.
   a. Describe the behavior of the translational kinetic energy K_T.
   b. Describe the behavior of the rotational kinetic energy K_R .
   c. Describe the behavior of the gravitational potential energy U_g .
   d. Describe the behavior of the total mechanical energy E.
   e. Write down the magnitude of the force of friction.

2. Repeat Step 1 for another value of y_{ref}, say the middle y-value of your data.

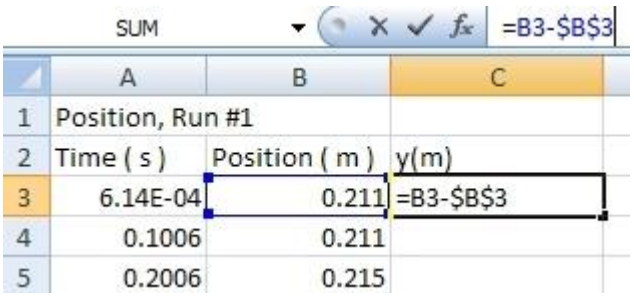
3. Save your Excel worksheet using **Save As → Excel Workbook** from File menu and **Send this file by email to your instructor as the following:**
Email Subject: your Serial Number- your Section Number
File name: your ID Number

4. Write down a suitable conclusion for this experiment.

APPENDIX

Using Excel to do the calculations

1. Type the heading y(m) in cell C2 and type the formula ***=B3-B3*** in cell C3 and ***Enter***. This makes the position y=0 at t=0.



2. To write other values of y, click on cell C3 and use the left mouse button to press on the small square at the lower right corner of the cell and drag it down to the last data to be entered in that column. Note that B3 remains the same in all the cells and therefore, it is called absolute reference. In this way, you make sure always s_0 is subtracted from *s* to find the corresponding *y*.

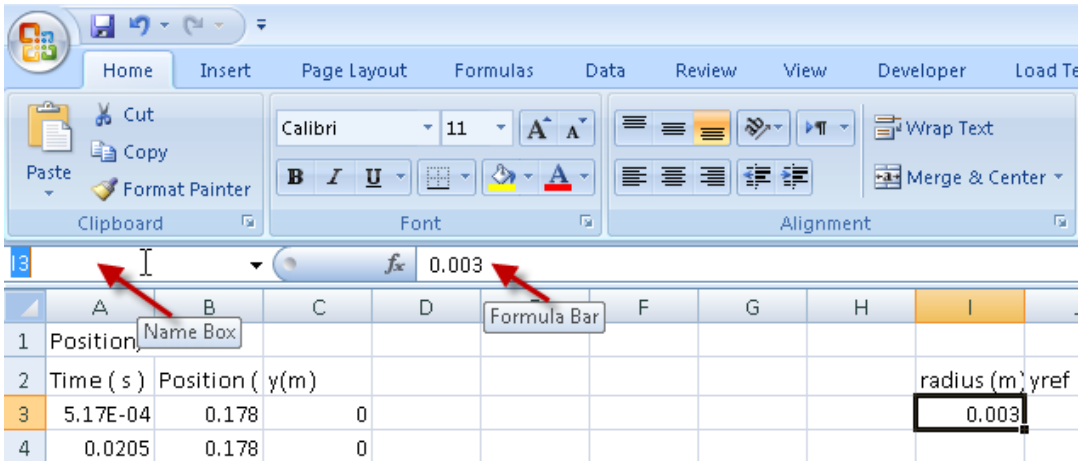
3. Return to step 5 in Exercise 2 of the write-up.

4. Type *radius (m)* in cell I2, y_{ref} *(m)* in cell J2, *m (kg)* in cell K2, *I (kg.m^2)* in cell L2, and *a (m/s^2)* in cell M2 as shown below. Also enter the values of m (0.450 kg) and I (0.00100 kg.m^2) in cells K3 and L3, respectively.

I	J	K	L	M
radius (m)	yref (m)	m (kg)	I (kg.m^2)	a (m/s^2)
		0.450	1.00E-03	

5. In cell *I3* type in <u>your measured value</u> of the radius (diameter/2) of the spindle in meters. In cell *J3* type in the last data of column C. This puts zero reference level for the potential energy at the end position of the Maxwell wheel. In cell M3 type in the value of *a* you found in Exercise 1.

6. Select cell *I3* and type *radius* in the ***Name box*** area and ***Enter***. This defines the value of radius to be the value in the cell I3.



7. Then select J3 and type *yref* in the ***Name box*** area and ***Enter***. This defines the value of y_{ref} to be the value in the cell J3. Similarly give the names *m*, *I* and *a* for the cells K3, L3 and M3, respectively.

8. Return to step 6 in Exercise 2 of the write-up.

9. Type in the headings in the 2nd row and the formulas in 3rd row of columns D, E, F and G as shown below:

D	E	F	G
K_t (J)	K_r (J)	U (J)	E (J)
=m*a*C3	=I*a*C3/radius^2	=m*9.8*(yref-C3)	=SUM(D3:F3)

10. To write other values of K_T, K_R, U_g, and E, select the four cells D3 to G3 in row 3 and use the left mouse button to press on the small square at the lower right corner of the cell G3 and drag it down to the last data to be entered.

11. Return to step 7 in Exercise 2 of the write-up.