Resonance on String

Objectives

a. Study of resonance phenomenon on a stressed string.
b. To find the value of acceleration due to gravity using the principle of resonance.
c. To apply error analysis principles to dependent quantities.

Apparatus Required

<table>
<tr><td></td><td></td><td rowspan="3"></td></tr>
<tr><td>String</td><td>Beam Balance</td></tr>
<tr><td></td><td>
</td></tr>
<tr><td>Weight Box</td><td>Sine wave generator</td><td>Mass Holder</td></tr>
</table>

Introduction

When two identical waves travel in opposite directions along a stretched string they interfere such a way that their resultant wave doesn't travel in space. Therefore, it is called a standing wave. Although the wave doesn't move in **standing wave** but the particles of the string vibrate in a unique way. In some points particles don't vibrate at all. Such points are called **nodes.** Whereas some points vibrate with maximum amplitude and they are called **antinodes**.

When a wave travels through a stretched string fixed at two ends it reflects back and forth from the fixed ends. At some specific frequencies, the original and reflected waves travel together in phase producing **standing wave** of large **antinodes**. Such conditions are called resonances or harmonics. The condition resonance to occur is

$$\lambda_n = \frac{2L}{n}. \qquad (1)$$

Where, n = number of harmonics, L = length of the string fixed at both ends λ_n = wavelength.

(Please refer appendix for more detail theory.)

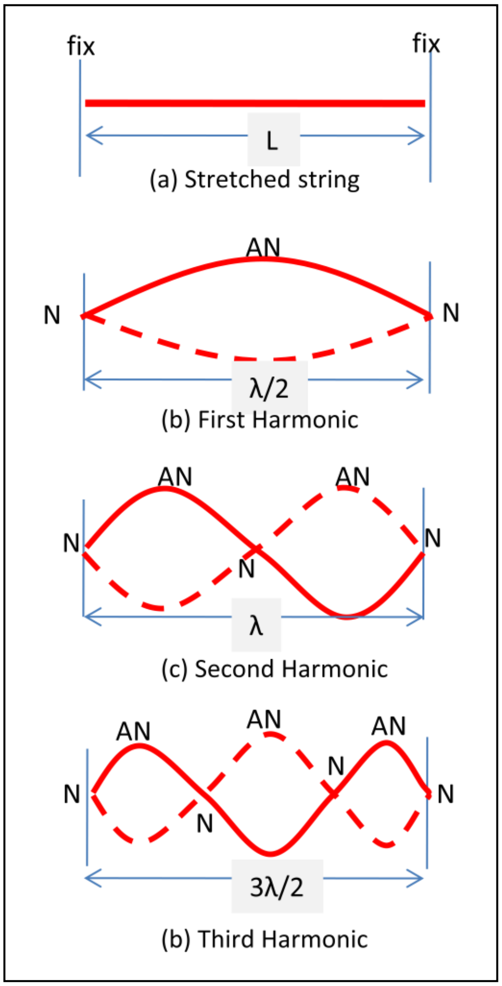

Figure 1: The schematic diagram of fundamental mode of vibration.

The first harmonic (n =1) takes place at λ_1 =2L and it will look like the schematic diagram in figure 1(b). Since the ends of the string are fixed, there must be nodes at the ends. Between two nodes there will always be an anti-node. The structure like figure 1(b) is called a loop or a segment. The length of each loop is always λ/2.

The second harmonic (n =2) will occur when λ_2 =L and the third harmonic (n =3) in λ_3 =2/3L.

But we know for the wave travelling in stretched string,

$$v = f\lambda \text{ and } v = \sqrt{\frac{\tau}{\mu}}$$

Where, μ= linear density of the string, v = velocity and f = frequency of the wave.

Therefore, above equation can be rewritten in terms of frequency and tension as:

$$f_n = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}} \qquad (2)$$

But tension is created by hanging the mass ***"m"*** under the action of gravity:

$$f_n = \frac{n}{2L}\sqrt{\frac{mg}{\mu}} \qquad (3)$$

The specific frequencies at which resonances or harmonics occur are called *resonant* frequencies. The lowest possible resonant frequency is called the *fundamental* frequency because the other higher frequencies at which resonance occurs are simply integral multiple of this frequency. In other word, if the fundamental frequency is f_1, the second resonance will take place at $2f_1$; third resonance will take place at $3f_1$ and so on. Hence the resonance frequency for n^{th} harmonics is:

$$f_n = nf_1 \qquad (4)$$

Part 1: Finding the value of *g*

In this section of the experiment you will be fixing the tension (mg) and get different harmonics (n) by changing frequency. By plotting the resonance frequency (f_n) v/s n you will obtain the average value of acceleration due to gravity g. In this experiment, you will be measuring μ, L and f_n.

Procedure:

1. You are provided with 4.0 m long string to measure the value of $\boldsymbol{\mu}$. Think of it how to measure its value. Discuss with your partner and confirm with your instructor. Calculate the value of $\boldsymbol{\mu}$ and record it. **Do you think the linear density of the 4.0 m string and the string on your experimental setup is different? Explain your answer.**

2. Your next job is to measure $\boldsymbol{L}$ and $\boldsymbol{m}$. For this purpose, you are provided with 1.5 m long string. Look at to the experimental set up. Attach the one end of the string to the vibrating blade (if it has not been attached already). Run the string over the pulley and hang any mass (higher than 50 g) to create tension.

   (Note: The length L is measured from the fixed end to the top of the pulley)

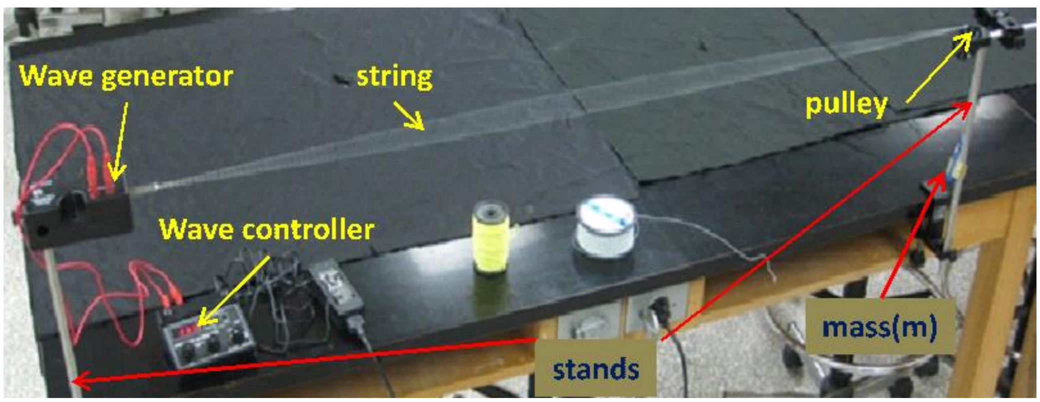

Figure 2: Real experimental set up with fundamental mode of resonance

3. Measure the value of L. It should be measured from knot at one end to the top of the pulley at the other end.

4. Start Excel sheet as shown in table 1. With the help of equation (3) predict the resonance frequencies (f_n) for different resonances (n).

n	μ(kg/m)	L(m)	$\Delta L(m)$	m(kg)	f_n-predicted (Hz)	f_n-observed (Hz)	$\Delta f(Hz)$	% error in f
1			Fill this value in part 2				Fill this value in part 2	
2								
3								
4								
5								

Table 1

Now you might get some idea that your job in this experiment would be to find the experimental value of resonance frequencies and compare them with the theoretically predicted values.

5. Turn on the Sine Wave Generator and turn the Amplitude knob all the way down (counter-clockwise). Connect the Sine Wave Generator to the string vibrator using two banana patch cords. Polarity does not matter.
6. Let the Amplitude knob be about midway. Use the Coarse (1.0) and Fine (0.1) Frequency knobs of the Sine Wave Generator to adjust the vibrations so that the string vibrates in *one* segment (n =1) as shown in Fig. 2. Adjust the driving amplitude and frequency to obtain a large-amplitude wave, but also check the end of the vibrating blade (the point where the string tied to the metal blade) which should be a node (see Fig. 2). **Fix the driving amplitude (preferably at lower values) so that the node at the <u>end of the vibrating blade</u> is right at the knot of the string**. It is more important to have a good node at the blade-end than it is to have the largest amplitude possible. However, it is desirable to have large amplitude while keeping a good node. Once you confirm the best looking first harmonics, record the frequency in the Excel table. Check if your observed value is closer to the predicted value.

7. Repeat step 6 for n = 2, 3, 4 and 5.

8. Plot the $\boldsymbol{f_n}$***-experimental*** against $\boldsymbol{n}$ in excel and find the value of g from the slope. Compare the experimental value with the standard value.

Part 2: Error Analysis:

The value of g that you measured in this experiment is being indirectly measured. The uncertainty (Δg) in its measurement depends on the uncertainty of directly measured quantities such as f, L and m. Uncertainty in the measurement of mass is quite minimal. Therefore, Δg mostly depends on ΔL and Δf.

From equation (3) the uncertainty in g for a specific harmonic can be derived as,

$$\Delta g = 2\left(\frac{\Delta L}{L} + \frac{\Delta f}{f}\right) g. \tag{5}$$

Please see the appendix 2 for more detail.

<u>**Procedure to measure ΔL and Δf**</u>

1. There should be minimal error on measuring the length of the string from the knot to the pulley top. However, while resonance occurs the node may not be right at the knot of the string as you can see the vibrating blade is quite flexible and sometime node can be seen beyond the knot somewhere in the vibrating blade. Considering the worst-case scenario, you can take the length of the blade as the uncertainty in L. The outside exposed length of the vibrating blade would be good enough approximation for ΔL.

2. To measure Δf**,** find the best value of fundamental frequency for the first harmonic again. Fix the amplitude of the source. Increase or decrease the frequency form the best resonance condition until you notice the change in amplitude. The frequency difference is your uncertainty (Δf) on measuring the frequency.

3. Find upper and lower limits ($g \pm \Delta g$) of g and conclude if your value lies within the range.

Tail Questions for report: (Instructor can add more questions by themselves if needed)

1. Slope of a straight line gives average value of the quantity at question. If you vary mass instead of frequency in part 1 of the experiment explain with equation that which two quantities will you plot in a straight line and what will be its slope?

2. If you are provided with a string thicker than your experimental string. How would it effect the fundamental frequency? Explain.

3. Look at the experimental set up carefully. Do you think you can vary length instead of frequency to get different resonances? Explain.

4. In your experimental set up for 4^{th} harmonic, if your mass was 500 g, what would be your frequency?

(Optional)

Part II: Find the fix number of harmonics (*n*) for different tension in the string

In this exercise you will fix the number of harmonics but vary mass to find the respective resonant frequency for the harmonic.

We can rewrite the equation (3) in the form.

$$f_n^{\,2} = \frac{n^2 g}{4\mu L^2} m \qquad (5)$$

1. Hang about 50 g of mass from the string over the pulley. Record the total hanging mass, including the mass hanger.

m(kg)	***μ(kg/m)***	*L*(m)	*n*	***f_n-predicted (Hz)***	***f_n-observed (Hz)***	*% error in f*
0.05			4			
0.10			4			
0.15			4			
0.20			4			
0.25			4			

Table 2:

2. Adjust the frequency of the Sine Wave Generator so that the string vibrates in *four* segments. As before, adjust the driving amplitude and frequency to obtain a large-amplitude wave, *and* clean nodes, including the node at the end of the blade. **Record the frequency.**

3. Repeat the experiment for different masses as shown in table 2 and make a excel table.

Note: For this part of the experiment, you will always adjust the frequency so that the wave vibrates in *four* segments.

For the forth harmonics the equation turns out to be:

$$f_4^{\,2} = \frac{4g}{\mu L^2} m$$

Note: You can use the theory to predict the frequency as in Exercise I.

4. Scatter plot f_n^2, v/s ***m*** with .

5. Find the slope of the curves with the linear line fit and calculate "g" with the help of the following equation.

$$slope = \frac{4g}{\mu L^2}$$

6. Find how much does it differ from the standard value

Appendix 1:

If a wave of angular frequency ω and wave vector k is travelling to the right through a stretched string fixed at both ends as shown in figure 1 (a) it can be represented by the following equation.

$$y = y_m sin(kx - wt) \qquad \text{(a-1)}$$

When the wave gets reflected from the boundaries and make a full round, it travels 2L distance more compare to the original wave. If the reflected wave returns to its original position in time period T, the equation can be written as:

$$y = y_m sin(k(x + 2L) - w(t + T)) \qquad \text{(a-2)}$$

Where, L is the length of the string between two fixed points. It should be noted that each time the wave reflects from the fixed boundary it undergoes phase change by π.

In such case the reflected wave and incident wave will be in phase, hence the first resonance will take place when:

$$k(x + 2L) - (\omega t + 2\omega T) - (kx - \omega t) = 0$$

$$k(2L) = 2\pi$$

But if the reflected wave returns to its original position in time period 2T, the equation can be written as:

$$y = y_m sin(k(x + 2L) - w(t + 2T))$$

In that case the condition of second harmonics will be:

$$k(2L) = 2(2\pi)$$

Similarly, the n^{th} harmonic will takes place when,

$$k(2L) = n(2\pi) \qquad \text{(a-3)}$$

Where, n = 1, 2, 3 ....n and is the number of harmonics.

Solving equation (a-3) for the wavelength ($\lambda_n = \frac{2\pi}{k}$) of the wave at n^{th} harmonics, we obtain,

$$\lambda_n = \frac{2L}{n} \qquad \text{(a-4)}$$

Appendix 2:

Taking square of the equation (3) and readjusting

$$g = \frac{4\mu}{mn^2} L^2 f^2 \tag{a-5}$$

Considering μ, n and m as constant.

$$\frac{\Delta g}{g} = 2\frac{\Delta L}{L} + 2\frac{\Delta f}{f} \tag{a-6}$$