## Capacitance

## Objectives

a. To learn the charging and discharging properties of a capacitor
b. To learn about the series combination of capacitors
c. To learn about the parallel combination of capacitors

## Apparatus Assembly



## Introduction

## Capacitance

A capacitor is a device that is used to store charge and its capacity to store charge is called capacitance. Mathematically, the capacitance $\mathbf{C}$, can be written as:

$$
\begin{equation*}
C=\frac{\mathbf{q}}{\mathbf{v}} \tag{1}
\end{equation*}
$$

Where $\mathbf{V}$ is the applied potential difference and $\mathbf{q}$ is the quantity of the charge stored in the capacitor.

## Charging of Capacitor

When an uncharged capacitor of capacitance $\mathbf{C}$ is connected to a battery of emf $\mathbf{V}_{\mathbf{0}}$ through a resistor $\mathbf{R}$ its voltage $\mathbf{V}$ increases with time and their relationship is given by the equation (2).

$$
\begin{equation*}
\mathbf{V}=\mathbf{V}_{\mathbf{0}}-\mathbf{V}_{\mathbf{0}} \mathbf{e}^{-\mathbf{t} / \tau} \tag{2}
\end{equation*}
$$

Where, $\tau=\mathrm{RC}$ is the time constant, defined as the capacitor's time to charge (discharge) by $63.2 \%$.

## Discharging of capacitor

When a fully charged capacitor with initial voltage $\mathbf{V}_{\mathbf{0}}$ is connected in a circuit through a resistor its voltage will decrease in time according to equation (3).

$$
\begin{equation*}
\mathbf{V}=\mathbf{V}_{\mathbf{0}} \mathbf{e}^{-\mathbf{t} / \tau} \tag{3}
\end{equation*}
$$

The process is called the discharging of the capacitor.
The current in the discharging circuit decreases in a similar manner as in the charging circuit.
Whether the process is charging or discharging, the current will exponentially increase from $t=0$. The relationship is provided in equation (4).

$$
\begin{equation*}
I=I_{0} e^{-t / \tau} \tag{4}
\end{equation*}
$$

## Parallel combination of capacitors

When $\boldsymbol{n}$ numbers of capacitors are connected in parallel each capacitor has the same amount of charge. The equivalent capacitance $\mathbf{C}_{\mathbf{p}}$ of parallel combination can be written as:

$$
\mathbf{C}_{\mathbf{p}}=\mathbf{C}_{\mathbf{1}}+\mathbf{C}_{\mathbf{2}}+------+\mathbf{C}_{\mathbf{n}}
$$

If there are only two resistors in parallel combination as shown in Fig.,

$$
\begin{equation*}
C_{p}=C_{1}+C_{2} \tag{5}
\end{equation*}
$$

## Series combination of capacitors

When $n$-number of capacitors are connected in series they shares the same potential difference. The equivalent capacitance $\mathbf{C}_{\text {s }}$ for series combination is given by the following formula:

$$
\frac{1}{C_{s}}=\frac{1}{C_{1}}+\frac{1}{C_{2}}+------+\frac{1}{C_{n}}
$$

If there are only two resistors connected in series as shown in Figure,

$$
\frac{1}{C_{s}}=\frac{1}{C_{1}}+\frac{1}{C_{2}}
$$

$$
\begin{equation*}
\mathrm{C}_{\mathrm{s}}=\frac{\mathrm{C}_{1} \mathrm{C}_{2}}{\mathrm{C}_{1}+\mathrm{C}_{2}} \tag{6}
\end{equation*}
$$

## Part 1: Finding

In this exercise you will learn to connect the charging and discharging circuits of capacitors from the circuit diagram. Then you will find the actual value of the capacitors from charging and discharging curves.


1. Take a connection board and identify the items in the circuit diagram. A voltmeter and ammeter are built into the Pasco I-V sensor. $\mathbf{C}$ is a capacitor, $\mathbf{R}$ is a resistor, and $\mathbf{V}_{0}$ is the voltage of the battery.
2. Make sure you are provided with two capacitors of different values, two resistors of different values, a Pasco-voltage sensor, a power supply and wires.
3. Take a bigger valued capacitor and name it as $\mathbf{C}_{1}$. Read out the voltage and capacitance of the capacitor. Never exceed the voltage limit written in the capacitor. If you ever exceed you will damage the capacitor.
4. A resistor of resistance $\mathbf{R}$ is provided to you. If it is not provided then you can use the external voltmeter.
5. Multiply the values of $\mathbf{R}$ and $\mathbf{C}_{1}$. Ensure the units of $\mathbf{R}$ and $\mathbf{C}$ are Ohms and Farad respectively. The result is called time constant and it can help you estimate the
discharging time. $63.4 \%$ of the capacitor voltages discharges during $\boldsymbol{\tau}=\mathbf{R} \times \mathbf{C}$ This value has to be a few seconds that can be comprehendible. For example: if the capacitor is $1000 \mu F$, the resistance is $10 \Omega$, then, $\tau=10 \times 1000 \times 10^{-6}=10^{-2} s$. It is a flash of second and such time cannot be comprehendible. But if the resistance is $10 \mathrm{k} \Omega$, $\tau=10 \times 1000 \times 1000 \times 10^{-6}=10 \mathrm{~s}$. Such time can be comprehendible. (Note that the sensor voltmeter also has a resistance. It is about a million Ohm. Such a high resistance in parallel does not contribute much. So, you can ignore its effect)
6. Connect the circuit diagram according to figure 1 using C 1 and R in the circuit.
7. Make sure the positive side of the capacitor as well as of the voltmeter are connected to the positive side of the battery.
8. Make sure points 1,2 , and 3 are open. They should not be connected to each other. A cable with a banana connector can be used as a two-way connector.
9. Don't turn on the power supply yet.
10. Connect the I-V sensors to the computer through the interface.
11. Open data studio/new experiment/graph
12. Set Voltage v/s time graph.
13. Turn on the power supply. Set the power supply voltage $\mathbf{V}_{\mathbf{0}}$ at any value between 5 to 10 V. Check on the capacitor label and make sure the labeled voltage on capacitor is greater than $V_{0}$.
14. Have your instructor verify your connection.
15. Start data studio. The voltage-time graph should read zero. If not, discharge the capacitor by directly connecting the two terminals of the capacitor.
16. Connect points 1 and 2 in the circuit diagram so that the capacitor will be connected to the battery and gets charged.
17. Once the voltage reading in the data studio saturates to $\mathbf{V}_{0}$ disconnect points 1 and 2 and connect 1 and 3. The voltage goes down and you get a discharged curve. Stop the software once the voltage turns to a flat line (almost zero V ). This means the capacitor is completely discharged. The charging and discharging curve would look like Figure 2.

18. Find the exponent of the curve by the exponential curve fitting. (Inverse Exponent Fit for charging curve and Natural Exponent Fit for discharging curve.)
19. The charging and discharging equations infer that the absolute value of exponent, $\boldsymbol{E}=$ $\frac{\mathbf{1}}{R C_{1}}$. Find the value of $\mathbf{C}_{\mathbf{1}}$ from the average value of exponents and fill in Table 1.
20. Calculate the theoretical value of voltage $\mathbf{V} \boldsymbol{\tau}$ at time $\boldsymbol{\tau}=\mathbf{R C}$ ( $63.2 \%$ of maximum voltage). Use the axes tool in the data studio to find the experimental value of $\mathbf{V} \boldsymbol{\tau}$ and fill in Table 1.
21. Measure the experimental value of $\mathbf{V} \boldsymbol{\tau}$ using coordinate tool and compare it with the theoretical value. (Find the voltage difference between time $t=0$ to $t=R C$ from either charging or discharging curve. Make sure you are taking $t=0$ while charging or discharging starts, not from when you run the time)

| Capacitor | $\mathrm{R}(\Omega)$ | E_ch | E_dch | E_av | $C=\frac{1}{R E_{a v}}$ <br> $(\mu F)$ | C-th <br> $(\mu F)$ | C <br> $\%$ error | V $\tau$ _ex <br> $(\mathrm{V})$ | V $\tau$ _th <br> $(\mathrm{V})$ | V $\tau$ <br> $\%$ \%error |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| C 1 |  |  |  |  |  | x | x |  |  |  |
| C 2 |  |  |  |  |  | x | x |  |  |  |
| Cs |  |  |  |  |  |  |  |  |  |  |
| CP |  |  |  |  |  |  |  |  |  |  |

Table 1: Data analysis summery
$\mathrm{E}_{\text {_ch }}=$ Absolute value of exponent from the charging curve
$\mathrm{E}_{-\mathrm{dch}}=$ Absolute value of exponent from the charging curve
$\mathrm{E}_{\text {_av }}=$ average value of exponent
$\mathrm{C}_{\text {_th }}=$ theoretical (expected value of C)
$\mathrm{V} \tau_{\_}$ex $=$experimental value of voltage while charging (discharging) happens from start time to $t=R C$.
$\mathrm{V} \tau_{-}$th $=0.632 \mathrm{~V}_{0}$.
22. You can get the value of exponent from current equation as well. However, the current is too small to be detected by our sensors.
Part 2: Finding the capacitance of second capacitor: $\mathbf{C}_{2}$

1. Replace capacitor $\mathbf{C}_{1}$ with the second capacitor $\mathbf{C}_{2}$ in the circuit. Delete all runs on Data studio.
2. Repeat steps 15 to 21 and fill in the result in Table 1.

## Part 3: Investigating series combination of capacitors.

1. Connect two capacitors in series as shown in Figure 3.
2. Delete all runs on Data studio.
3. Repeat steps 15 to 21 and fill in the result in Table 1.
4. Find expected value from the theory equation (6).


Part 4: Investigating the pparallel combination of capacitors.

1. Connect two capacitors in parallel as shown in Figure 4.
2. Delete all runs on Data studio.
3. Repeat steps 15 to 21 and fill in the result in Table1.
4. Find expected value from the theory equation (5).

