# Experiment 1
# Data Analysis and Presentation

## Purpose

To learn how to analyse experimental data and to practise error analysis.

### Exercise 1

The data in the following table relates to measurements made of the period of oscillation $T$ of a simple pendulum of variable length $L$. Theory predicts the following relationship:

$$T = 2\pi\sqrt{\frac{L}{g}} \quad (1)$$

where $g$ is the acceleration due to gravity.

**Table 1. Your instructor may give you a different set of data than this during the lab session.**

| Length ($L$) cm $\pm$ 0.5 cm | 40.1 | 34.8 | 30.3 | 24.8 | 20.2 | 14.9 | 10.1 |
|---|---|---|---|---|---|---|---|
| Period ($T$) sec $\pm$ 0.05 sec | 1.28 | 1.17 | 1.13 | 0.98 | 0.92 | 0.76 | 0.66 |
| $g$ (cm/s$^2$) | | | | | | | |

1. Copy Table 1 and calculate a value of $g$ for each data point using equation (1). Find the average of $g$ for all data points. Call this value $g_1$. <u>Observe the rules of significant figures when doing the calculation</u>.

   $g_1 =$

2. Look at the $g$ values in Table 1 and make a rough estimate for the possible error in $g$ (call it $\Delta g_1$) from the range or spread of the data, i.e.

   $$\Delta_{g1} = \frac{g_{max} - g_{min}}{2} =$$

3. **3.** Record your experimentally determined value of $g$ as $g_1 \pm \Delta g_1$

4. Write down the general expression for the fractional maximum possible error (MPE) (relative uncertainty in g) $\Delta g/g$ in terms of $\Delta T/T$ and $\Delta L/L$. Read the Appendix.

   $\Delta g/g =$

5. Calculate $\Delta g_2$ for one of the data point (call it $g_2$), say for $L$ = 40.1 cm, from the table in step (1) and using the equation from step (4).

   $\Delta g_2 =$

6. Record this value of $g$ (from single data at $L$ = 40.1 cm ) with its error as $g_2 \pm \Delta g_2$

7. Using the data supplied construct another table as follows:

   Prove that $\Delta(T^2) = 2T\Delta T$ .

**Table 2.**

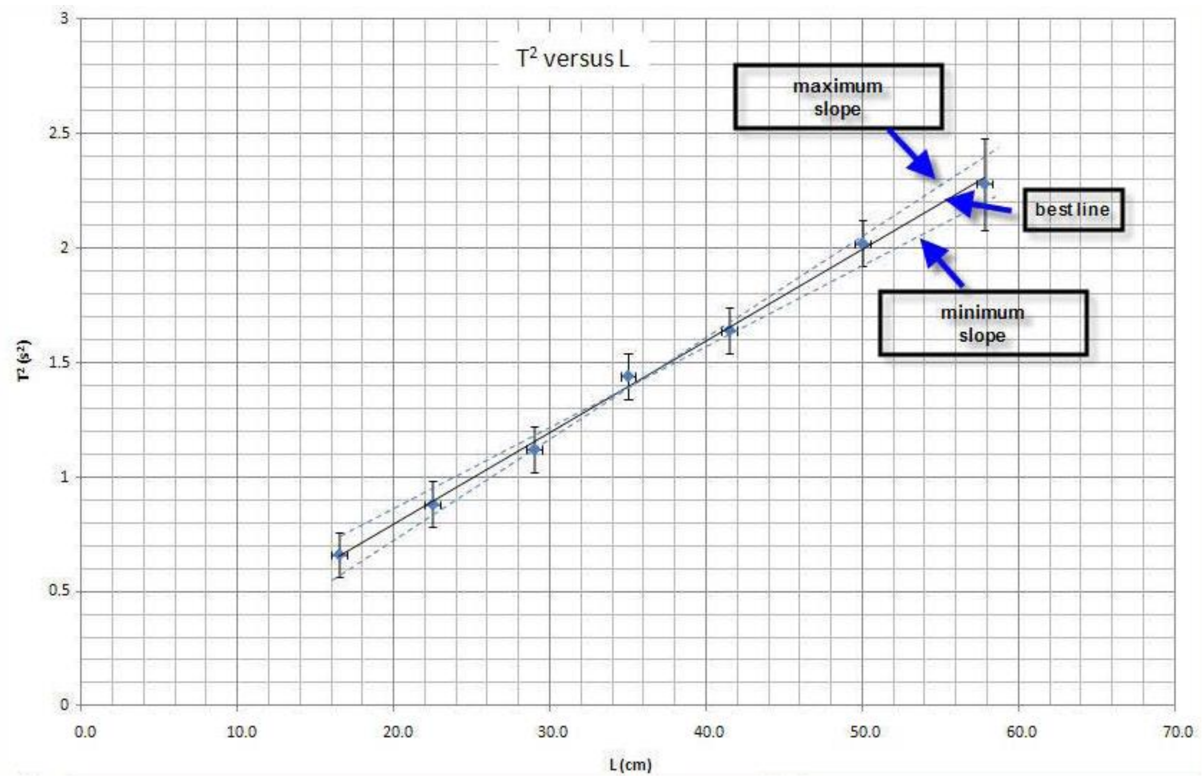
| Length ($L$) cm | Period ($T$) sec | $T^2$ sec$^2$ | $\pm \Delta L$ cm | $\pm \Delta T$ sec | $\pm \Delta (T^2) = 2T\Delta T$ sec$^2$ |
|---|---|---|---|---|---|
| 40.1 | 1.28 | | | | |
| 34.8 | 1.17 | | | | |
| 30.3 | 1.13 | | | | |
| 24.8 | 0.98 | | | | |
| 20.2 | 0.92 | | | | |
| 14.9 | 0.76 | | | | |
| 10.1 | 0.66 | | | | |

Use the data in this table to plot $T^2$ versus $L$ **in graph paper**. Show the best-fit straight-line of your data points and determine $g_3$ from the slope. Make sure you write a title for the graph and label the x-axis and y-axis with their units. Chose a convenient scale so that your plot fills the graph paper. To get accurate slope mark two distant points (far apart) on the line and determine their $x$-axis and $y$-axis values ($x_1$, $y_1$) and ($x_2$, $y_2$) (make sure $x_2 > x_1$). Show the values of $\Delta$y and $\Delta$x on your graph and calculate the slope**.**

Slope = $\Delta y/\Delta x = (y_2 - y_1) / (x_2 - x_1)$.

8. Include error bars and determine $\Delta g$ from your graph.

**Hint:** To find $\Delta g_{slope}$, find the maximum and minimum slopes and calculate the corresponding values of $g_{min}$ and $g_{max}$.




Maximum Slope = $g_{min}$=

Minimum Slope = $g_{max}$=

Average Slope = $g_3$ =

$$\Delta g_{slope} = \frac{\text{maximum g - minimum g}}{2}$$

9. Record this value of $g$ ($g_3$) with its uncertainty as $g_3 \pm \Delta g_{slope}$.

10. Take the *accepted* value of $g$ in Dhahran area as $g_{Dh}$ = 980 ± 1 cm/s$^2$. Fill the table below and observe how the three different values of $g$ agree (or disagree) with the *accepted* value and with each other within their uncertainty limits. Put a *tick* mark if the range overlaps with the range of *accepted* value or a *cross* mark if it doesn't, in the last column.

| | $g$ (cm/s$^2$) | $\Delta g$ (cm/s$^2$) | % error ($\Delta g/g$) ×100 | lower bound (cm/s$^2$) | upper bound (cm/s$^2$) | agree with *accepted*? |
|---|---|---|---|---|---|---|
| *accepted* | 980 | 1 | 0.1 | 979 | 981 | |
| from step (3) | | | | | | |
| from step (6) | | | | | | |
| from step (8) | | | | | | |

**Exercise 2**
Two lengths are measured with a meter ruler X = 35.5 cm and Y = 67.3 cm, where the uncertainty is $\Delta X = \Delta Y = 0.2$ cm .

1. Find the percentage error in $Z_1$, where $Z_1 = X + Y$.
2. Find the percentage error in $Z_2$, where $Z_2 = X - Y$.
3. Find the percentage error in $Z_3$, where $Z_3 = \frac{X}{Y}$.
4. Find the percentage error in $Z_4$, where $Z_4 = XY$.
5. Find the percentage error in $Z_5$, where $Z_5 = X^2/Y^3$.

## APPENDIX

### 1. Simple Error Analysis

Suppose that a quantity $X_o$ is measured. It can usually be stated that the uncertainty in the measurement lies within some 'reasonable maximum range',

$$\text{i.e. } X_o \pm \Delta X$$

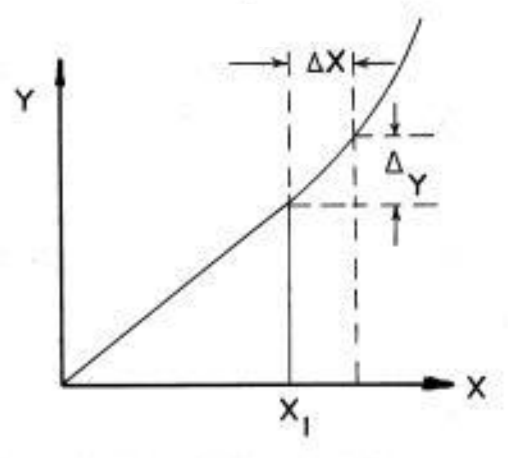
where $\Delta X$ is called the 'maximum possible error' MPE (uncertainty in $X_o$), $\frac{\Delta X}{X_o}$ is called the fractional error (relative error) in $X_o$ and $\{(\frac{\Delta X}{X_o})\ X\ 100\ \}$ is the percentage error.

### 2. Error Calculations and Differential Calculus

In the past you used to express errors as percentage errors when dealing with experimental measurement. In general, now that you have learned the elements of calculus, you will find that it is much easier to derive results by differentiation. We will show this by examples that: $\Delta Y \approx \frac{\partial Y}{\partial X} \Delta X$ .

Example (1) : Simple Algebraic Relation

Suppose $Y = X^2$ where X is measured in an experiment and Y is calculated. Suppose there is an error $\Delta X$ associated with the observed value $X_1$. Then there is a corresponding error $\Delta Y$ in the calculated value of Y (see diagram). If $\Delta\overline{X}$ (and $\Delta\overline{Y}$) are small enough, we can treat them as differential quantities.




(This assumes that Y is reasonably smooth and continuous function, which is always true in experimental Physics).

By differentiation we obtain

$$\frac{dY}{dX} = 2X$$

When the changes are definite, we can write

$$\Delta Y = 2X \ \ \Delta X$$

$$\frac{\Delta Y}{Y} = \frac{2X \ \Delta X}{X^2} = 2\,\frac{\Delta X}{X} = 2 \text{ (fractional error in X)}$$

Example (2) : General Algebraic Relation

Let $Y = A\,X^n$

where A is a constant and n can be negative or a fraction.

Differentiating $\frac{dY}{dX} = A\,n\,X^{n-1}$

$$\Delta Y = A\,n\,X^{n-1}\ \Delta X$$

$$\frac{\Delta Y}{Y} = \frac{A\,n\,X^{n-1}}{A\,X^n}\,\Delta X = n\left\{\frac{\Delta X}{X}\right\}$$

Fractional error in Y = | n | (Fractional error in X)

For function of several variables

If Z = f (X,Y) is a continuous function of X and Y then: $\Delta Z \approx \frac{\partial Z}{\partial X}\Delta X + \frac{\partial Z}{\partial Y}\Delta Y$

Example (3): Functions of Several Variables

Consider the case where Z is a function of two independent variables A and B, i.e. Z = Z (A, B). The expressions for $\Delta Z$ for some common relations between Z and A, B are given in the following table:

| **Relation between Z and A, B** | **Relation between errors** |
|---|---|
| $Z = A + B$<br>$Z = A - B$ | $\Delta Z = \Delta A + \Delta B$ |
| $Z = AB$<br>$Z = A/B$ | $\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$ |
| $Z = X^n Y^m$ | $\frac{\Delta Z}{Z} = n\frac{\Delta X}{X} + m\frac{\Delta Y}{Y}$ |