## Density

## Objective

To determine the density of solids and liquids using different methods.

## Introduction

Density $\rho$ of a material is defined as mass per unit volume. That is,

$$
\begin{equation*}
\rho=\frac{\mathrm{m}}{V} \tag{1}
\end{equation*}
$$

where $m$ is the mass and $V$ is the volume of the object. In this lab, you will measure the density of materials using different methods.

## Part 1 - Density of solids by direct method

In this exercise, you will determine the density of solid disks made of different materials, shown in Figure 1, and compare with the accepted values.


Figure 1
For a regular-shape object such as a disk, its volume $V$ can be determined by measuring its diameter $2 r$ and height $h$ (see Figure 2), and then using the formula


Figure 2

1. Measure the mass $m$ of the aluminum disk using a triple-beam balance. Make sure the disk is dry.
2. Measure the diameter $2 r$ and height $h$ of the disk using a digital caliper as you did in the lab Significant Figures.
3. Calculate its volume $V$ and density $\rho$, using Equations (2) and (1), respectively.
4. Determine the percent difference between your experimental value and the accepted value of $\rho$, using the formula

$$
\text { Percent difference }=\frac{\mid \text { Experimental value }- \text { Accepted value } \mid}{\text { Accepted value }} \times 100
$$

5. Repeat Steps 1 to 4 for the other disks made of brass and Plexiglass.
6. Record your results in your report in the following format.

| Material | $\mathbf{m}(\mathrm{kg})$ | $\mathbf{2 r}(\mathrm{m})$ | $\mathbf{h}(\mathrm{m})$ | $\mathbf{V}\left(\mathbf{m}^{\mathbf{3}}\right)$ | $\boldsymbol{\rho e x p}\left(\mathrm{kg} / \mathrm{m}^{\mathbf{3}}\right)$ | $\boldsymbol{\rho \text { acc } ( \mathrm { kg } / \mathrm { m } ^ { \mathbf { 3 } } )}$ | \% difference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Aluminum |  |  |  |  |  | 2700 |  |
| Brass |  |  |  |  |  | 8400 |  |
| Plexiglass |  |  |  |  |  | 1180 |  |

## Part 2 - Density of liquids by direct method

In this exercise, you will determine the density of water and oil.

1. Measure the mass $m_{1}$ of an empty graduated cylinder labeled water, see Figure 3, using the triple-beam balance.


Figure 3: Two 100 ml graduated cylinders to be used for water and oil separately
2. Fill the graduated cylinder with water between 90 and 100 ml marks. Record the volume reading in $\mathrm{m}^{3}$. Note that $1 \mathrm{ml}=1 \mathrm{~cm}^{3}=10^{-6} \mathrm{~m}^{3}$.
3. Measure the mass $m_{2}$ of the graduated cylinder with water using the triple-beam balance.
4. Calculate the mass of water ( $m_{2}-m_{1}$ ).
5. Calculate the density of water using Equation (1).
6. Repeat steps 1 to 5 for oil using a different graduated cylinder (the one labeled oil).
7. Record your results in your report in the following format.

| Liquid | Water | oil |
| :--- | :---: | :---: |
| Mass of graduated cylinder, $\mathrm{m} 1(\mathrm{~kg})$ |  |  |
| Mass of liquid+graduated cylinder, $\mathrm{m} 2(\mathrm{~kg})$ |  |  |
| Therefore, mass of liquid, $\mathrm{m}=(\mathrm{m} 2-\mathrm{m} 1)(\mathrm{kg})$ |  |  |
| Volume of liquid, $\mathrm{V}\left(\mathrm{m}^{3}\right)$ |  |  |
| Therefore, density of liquid, $\rho\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ |  |  |

## Part 3 - Density of oil using U-tube

In this exercise, you will determine the density of oil using a U-tube and the concept of pressure. The experimental set up is shown in Figure 4. Since the water in the U-tube is in static equilibrium, pressures at points in the water at the same horizontal level must be the same. Therefore, pressure at the oil-water interface is equal to pressure at point B . This leads to

$$
\begin{equation*}
\rho_{\text {oil }}=\frac{\mathrm{h}_{\mathrm{w}}}{h_{\text {oil }}} \rho_{\mathrm{w}} \tag{3}
\end{equation*}
$$



Figure 4

## Proof of Equation (3)

$P_{\text {interface }}=P_{o}+P_{\text {oil }}$
$\mathrm{P}_{\text {oil }}$, the pressure due to the oil column above the interface can be written in terms of the weight of that column $\mathrm{W}_{\text {oil }}$ and the cross sectional area $\mathrm{A}_{\text {oil }}$ of that column as:

$$
\begin{aligned}
\mathrm{P}_{\text {oil }} & =\frac{\mathrm{W}_{\text {oil }}}{\mathrm{A}_{\text {oil }}}=\frac{\mathrm{m}_{\text {oil }} g}{\mathrm{~A}_{\text {oil }}}=\frac{\left(\rho_{\text {oil }} \mathrm{V}_{\text {oil }}\right) g}{\mathrm{~A}_{\text {oil }}} \\
& =\frac{\rho_{\text {oil }}\left(\mathrm{A}_{\text {oil }} \mathrm{h}_{\text {oil }}\right) g}{\mathrm{~A}_{\text {oil }}}=\rho_{\text {oil }} \mathrm{h}_{\text {oil }} g
\end{aligned}
$$

That is

$$
\mathrm{P}_{\mathrm{oil}}=\rho_{\text {oil }} \mathrm{h}_{\mathrm{oil}} g
$$

Similarly,
$\mathrm{P}_{\mathrm{B}}=\mathrm{P}_{\mathrm{o}}+\mathrm{P}_{\mathrm{w}}$ and $\mathrm{P}_{\mathrm{w}}=\rho_{\mathrm{w}} \mathrm{h}_{\mathrm{w}} g$


Since the pressure at any point in a given liquid at rest depends only on the depth,

$$
P_{B}=P_{\text {interface }} \rightarrow P_{\text {oil }}=P_{w} \quad \rightarrow \quad \rho_{\text {oil }} h_{\text {oil }}=\rho_{w} h_{w}
$$

1. Pour water into the U-tube until it is half filled. Then pour oil in the left arm as shown in Figure 4. This step might have already been done for you.
2. Record the position readings at the top of oil column $y_{o i l}$, at the top of water column $y_{w}$, and at the interface $y_{B}$.
3. Calculate the heights $h_{o i l}$ and $h_{w}$ from the position readings.
4. Calculate the density of oil $\rho_{\text {oil }}$ using Equation (3) and the value of $\rho_{\text {water }}$ you obtained in Exercise 2.
5. Determine the percent difference between the values of $\rho_{\text {oil }}$ you obtained in Exercise 2 and Exercise 3.
6. Record your results in your report in the following format.

| $y_{\text {oil }}$ |  | $(\mathrm{m})$ |
| :--- | :--- | :--- |
| $y_{\mathrm{W}}$ |  | $(\mathrm{m})$ |
| $y_{\mathrm{B}}$ |  | $(\mathrm{m})$ |
| $h_{\text {oil }}$ |  | $(\mathrm{m})$ |
| $h_{\mathrm{W}}$ |  | $(\mathrm{m})$ |
| $\rho_{\text {oil }}$ |  | $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ |
| $\%$ diff |  |  |

## Part 4 - Density of solids by Archimedes' principle

In this exercise, you will use Archimedes' principle to determine the density of the three disks used in Exercise 1. You will also determine the density of an irregular-shape object using Archimedes' principle. Note that finding the volume of an irregular-shape object is not that straight forward. In this case, Archimedes' principle becomes handy to determine the density.

Archimedes' principle states that if a body is immersed in a fluid, it experiences an upward buoyant force $B$ equal to the weight of the fluid displaced by the body.

If the object is completely immersed in water, then the volume of the displaced water is equal to the volume of the body itself. Therefore,

$$
\begin{equation*}
B=\rho_{w} V g \tag{4}
\end{equation*}
$$

where $\rho_{w}$ is the density of water and g is the free-fall acceleration. Note that the buoyant force depends on the volume of the object, not its mass.

The free body diagram of the immersed object is shown in Figure 5.


Figure 5
Because of the buoyant force, the weight of the object inside water would appear to be less than its actual weight $W$. This is called apparent weight $W_{a}$. That is,

$$
\begin{equation*}
W_{a}=W-B \tag{5}
\end{equation*}
$$

Therefore,

$$
\begin{equation*}
B=m g-m_{a} g \tag{6}
\end{equation*}
$$

where, $m_{a}$ is the apparent mass. Substituting this in Equation (4) leads to

$$
\begin{equation*}
V=\frac{\left(\mathrm{m}-\mathrm{m}_{\mathrm{a}}\right)}{\rho_{\mathrm{w}}} \tag{7}
\end{equation*}
$$

Substituting this in Equation (1) leads to

$$
\begin{equation*}
\rho=\frac{\mathrm{m}}{\left(m-m_{a}\right)} \rho_{w} \tag{8}
\end{equation*}
$$

1. Measure the actual mass $m$ of the irregular-shape object, as you did in Exercise 1 for the disks, using a triple-beam balance. Make sure the object is dry.
2. Measure the apparent mass $m_{a}$ of the irregular-shape object by completely immersing it in the beaker of water, as shown in Figure 6, and re-adjusting the balance. Note that Equation (4) is valid only for complete immersion.


Figure 6
3. Calculate its density $\rho$ using Equation (8) and the value of $\rho_{w}$ you obtained in Exercise 2.
4. Repeat Steps 1 to 3 for the three disks used in Exercise 1.
5. Determine the percent difference between your experimental value and the accepted value of density.
6. Record your results in your report in the following format.

| Object | $\mathbf{m}(\mathbf{k g})$ | $\mathrm{ma}_{\mathrm{a}}(\mathbf{k g})$ | $\rho_{\mathrm{w}}\left(\mathbf{k g} / \mathbf{m}^{\wedge} \mathbf{3}\right)$ | $\boldsymbol{\rho}\left(\mathbf{k g} / \mathbf{m}^{\wedge} \mathbf{3}\right)$ | $\mathbf{\rho a c c}\left(\mathbf{k g} / \mathbf{m}^{\wedge} \mathbf{3}\right)$ | \% difference |
| :--- | :--- | :--- | :--- | :---: | :---: | :---: |
| Irregular |  |  |  |  | - | - |
| Aluminum |  |  |  |  | 2700 |  |
| Brass |  |  |  |  | 8400 |  |
| Plexiglass |  |  |  |  | 1180 |  |

7. List the major sources of error in this experiment (parts 1 through 4).
8. Write down an appropriate conclusion for this experiment
