The Spring

Objectives

1. To determine the force constant of a spring.

2. To study the conservation of mechanical energy in a spring-mass system.

Introduction

When a spring is pulled, the restoring force F_s of the spring is proportional to the extension x of the spring. This is known as Hooke's law. That is,

$$F_s = -k\,x$$

The negative sign indicates that the restoring force is opposing the extension, and the proportionality constant k is called the force constant of the spring or spring constant. It is a measure of stiffness of the spring; a stiffer spring has higher value of k.

A mass hanger of mass m is hung from the spring as shown in Figure 1b. Two forces act on the hanger, the gravitational pull F_g $(= mg)$, and the spring force F_s. When the block is at rest the forces on it are balanced. That is,

$$\left| F_g \right| = \left| F_s \right|$$

$$m\,g = k\,x = k\,(y_0 - y_{eq})$$

$$y_{eq} = -\frac{g}{k}m + y_0$$

Note y is the position as measured by the motion sensor sitting at the bottom and x refers to the extension of the spring.
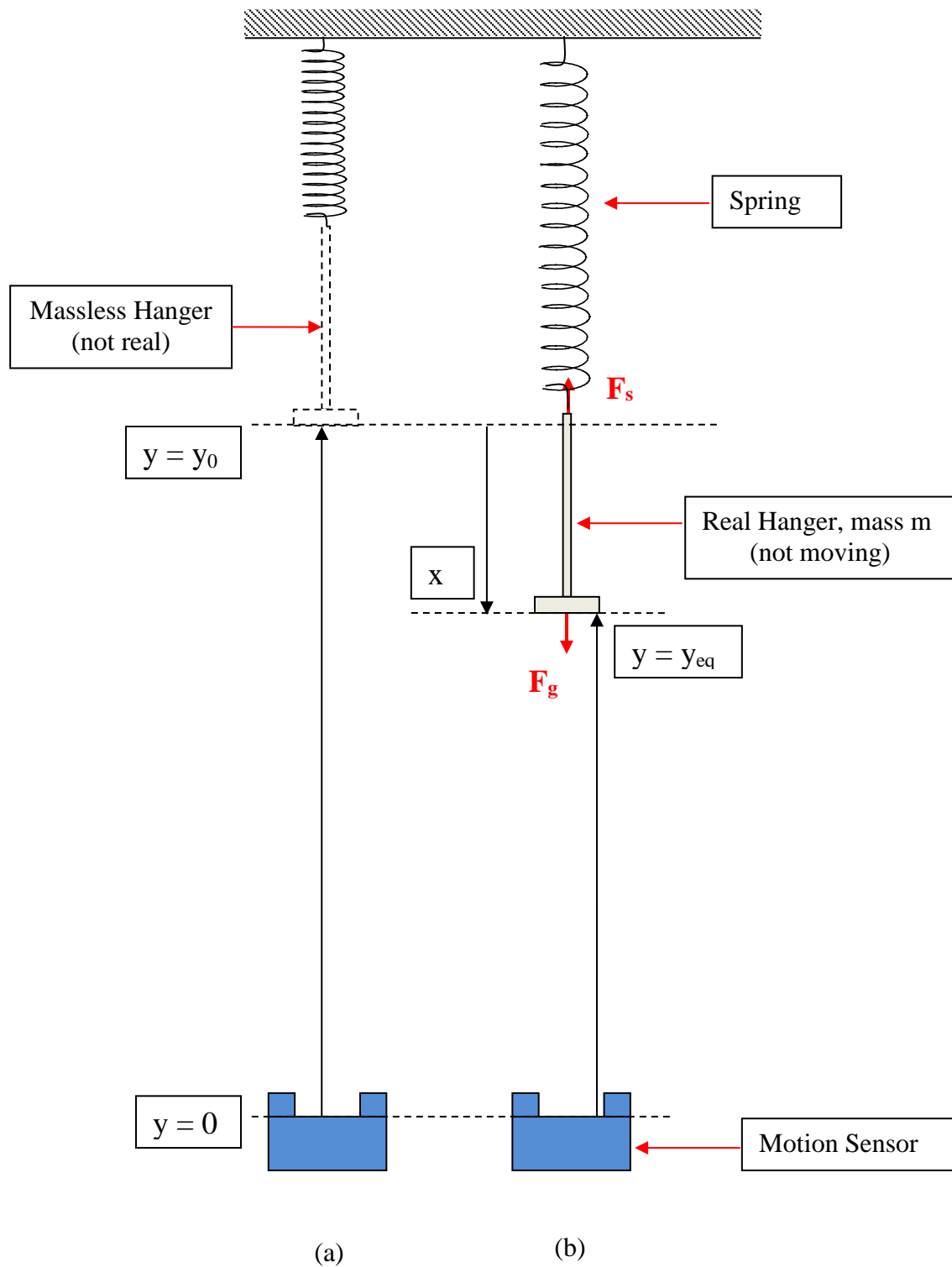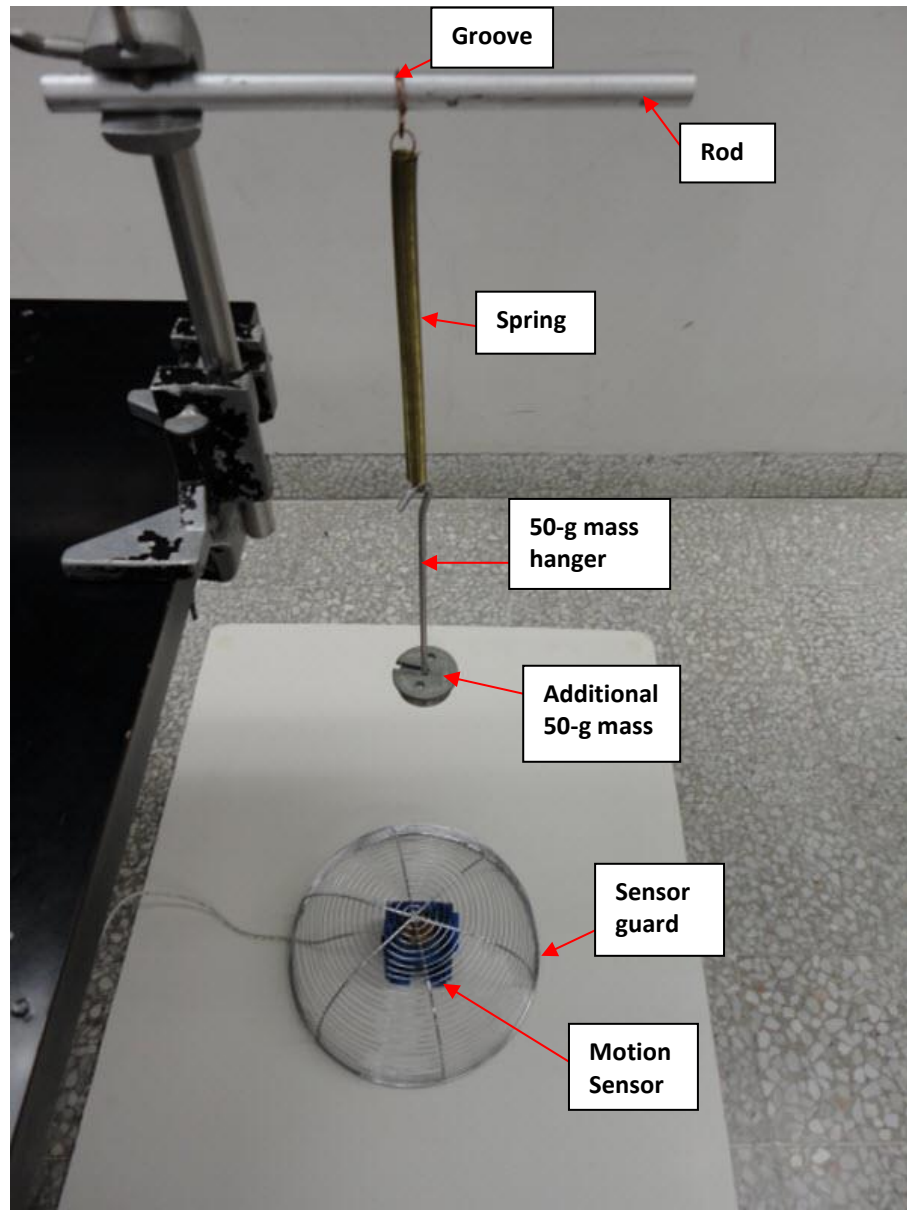
Figure 1

Exercise 1 – Determination of force constant *k*

1. As shown in Figure 2, hang the spring from the rod. Use the groove on the rod to keep the spring from moving horizontally. Hang a 50 g mass hanger from the free end of the spring and add an additional 50 g mass, this will make m =100 g.

Figure 2

2. Make sure the motion sensor is connected to a computer through the USB interface, and its range switch is set to work with the ⎓ (short range) option (see Figure 3).

3. Place the motion sensor on the floor and align it vertically with the mass hanger. Also make sure the *Ultrasonic Transducer* is parallel to the bottom face of the mass hanger. You can use the *Rotating Head* to adjust the angle of the sensor, if needed. This alignment is very important for getting good results.
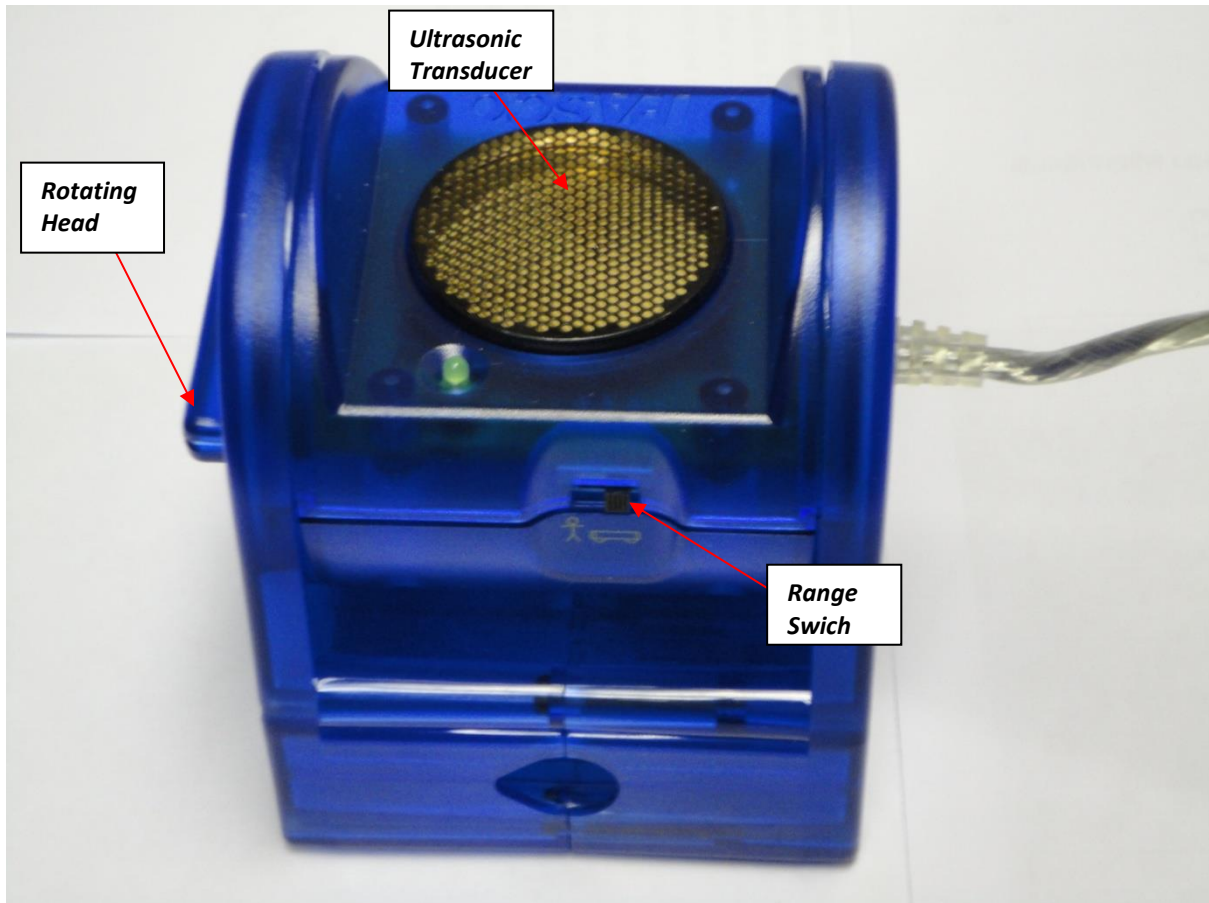
Figure 3

4. Download the file **spring.ds** from the link in phys101 homepage and save it on the desktop. This file has been preconfigured for optimum experimental parameters such as position and time scales.

5. From the **Start** button, go to **All Program → DataStudio → English**. This will open **DataStudio**. Click on *Open Activity*, navigate to the folder desktop and to the file **Spring.ds**, and open it. This will open the position-time graph.

6. Make sure that the spring-mass system is at rest, and then click the *Start* button in the **DataStudio** program. After about 2 seconds click the *stop* button.

7. Repeat the experiment for the following masses WITHOUT deleting the previous run, and continue to collect data in a similar manner: (the instructor will assign each group one of the following sets of masses)

   a. m = 120, 140, 160, 180 and 200 g
   b. m = 125, 145, 165, 185 and 205 g
   c. m = 130, 150, 170, 190 and 210 g

8. The enabled *Statistics Tool* will display the minimum, maximum and mean values of y_{eq} for all six runs in a table. Record the mean values in Table 1 of your report.

If you want to repeat the experiment for a particular mass m that you have just done, you can delete ONLY the last data run and repeat.

9. After completing all 6 runs, your data will look like Figure 4.

10. You can adjust the scales of your graph to get the best display by clicking on the **Settings** button ( ) and choosing the tab **Axis Settings** in it. Then type the appropriate minimum and maximum values for **position**. For example, since the data in Figure 3 is between 0.43 and 0.47 the appropriate minimum and maximum values have been chosen to be 0.35 and 0.55. Click **OK**.

11. Record the **mean** values of each run against the mass in a Table in your report.

Hanging mass m (kg)	Mean y value y_{eq} (m)

12. Plot y_{eq} versus m in your notebook.
13. Find the slope of the resulting straight line.
14. What does the slope represent?
15. Calculate the value of k depending on the slope you found above.
16. Determine the y-intercept.
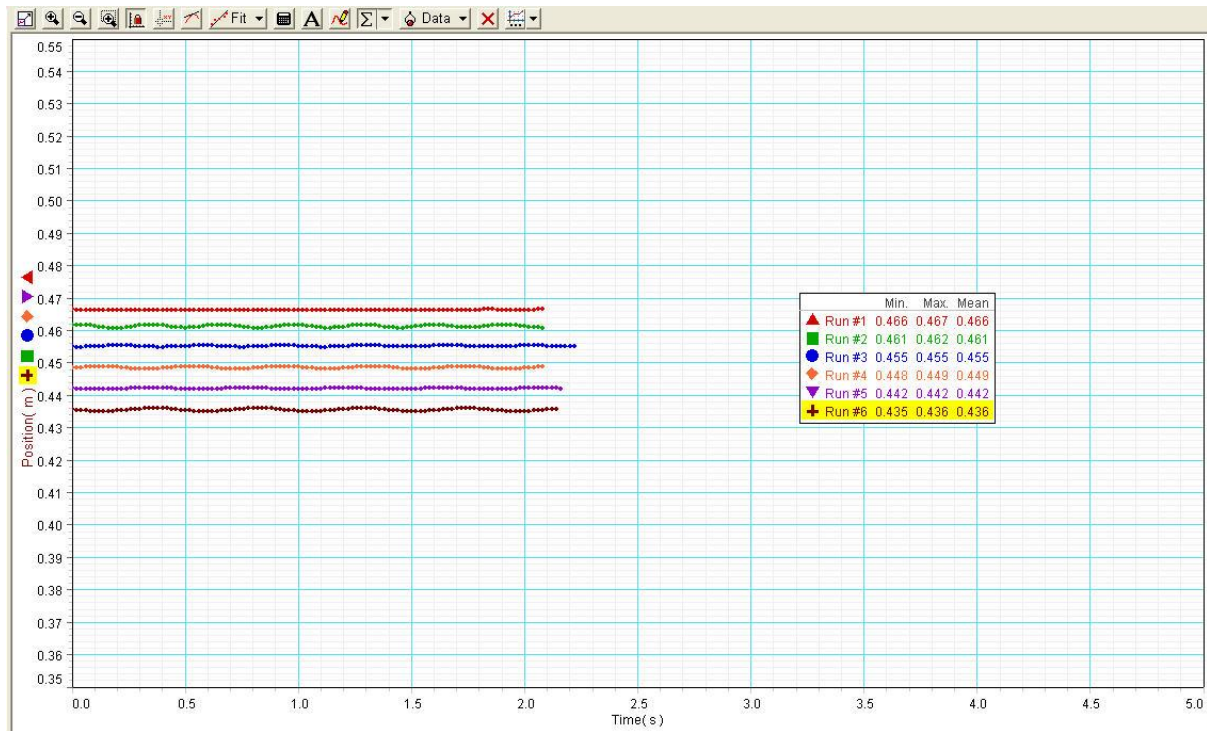17. What does the y-intercept represent?
18. Calculate the value of y_0.

Figure 4

Exercise 2 – Conservation of Energy

If only conservative forces do work within an isolated system, then the mechanical energy of the system cannot change. The mechanical energy is the sum of kinetic energy and potential energy of the system. In a spring-mass system, there are two conservative forces; spring's restoring force as well as gravitational force. The corresponding potential energies are given by:

$$U_S = \frac{1}{2} k x^2$$

$$U_g = m g y$$

The kinetic energy of the mass moving with velocity v is given by:

$$K = \frac{1}{2} m v^2$$

In this exercise, you will verify the law of conservation of energy.

1. Do not change the set up you used in Exercise 1. This is important because you will have to use the value of y_0 obtained from Exercise 1 for your calculations.

2. Delete all the data runs from the **DataStudio** file and get it ready for Exercise 2. Also right click in the graph area and un-tick *Statistics*.

3. Attach a 200 g mass to the same mass hanger that was used in Exercise 1 so that the total mass hanging from the spring is 250 g.

4. Pull the mass slightly down and let it go. This will make the spring-mass system to oscillate between the positions y_{min} and y_{max} as shown in Figure 5. Make sure this oscillation is only along the vertical line.

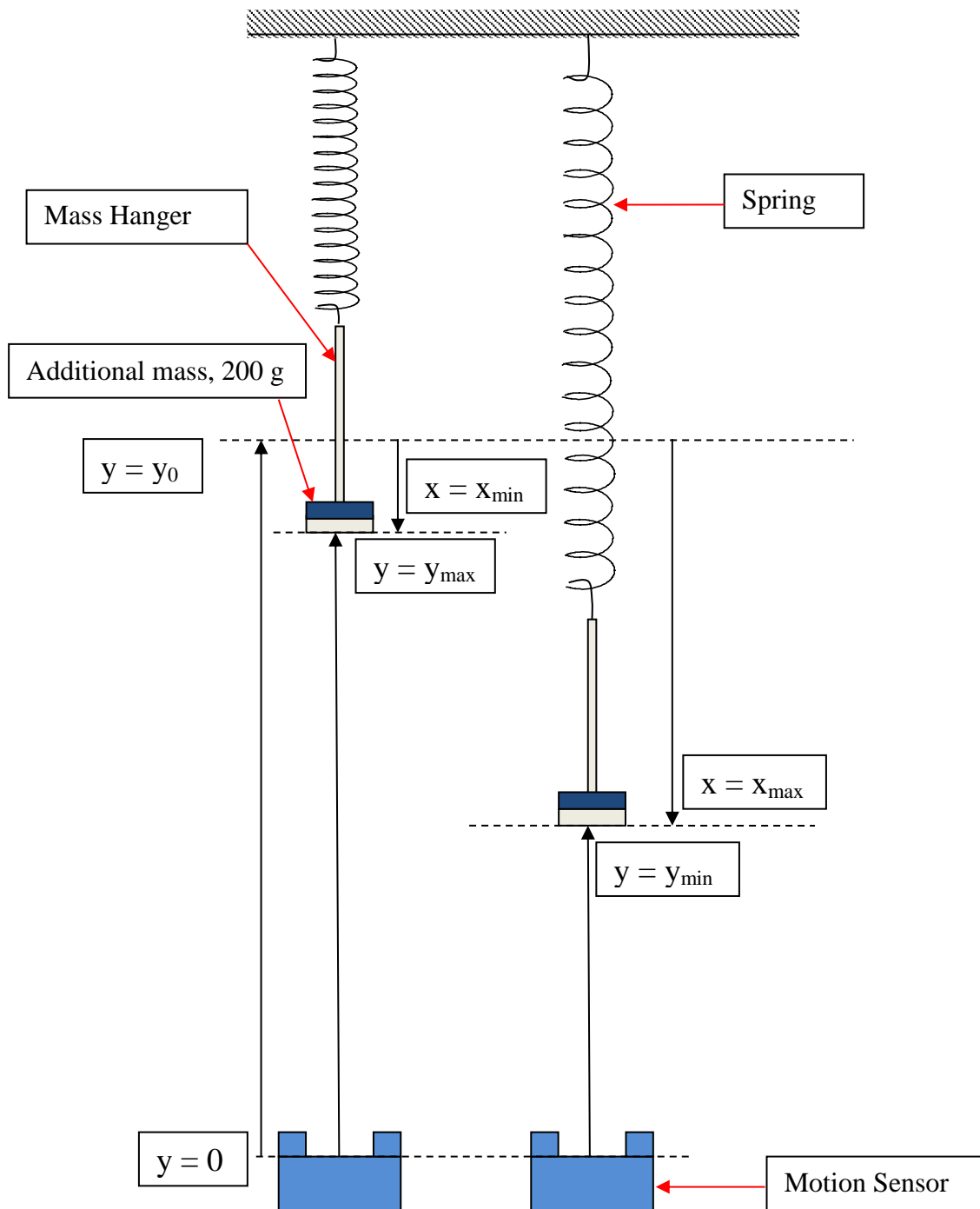Mass Hanger

Spring

Additional mass, 200 g

$y = y_0$

$x = x_{min}$

$y = y_{max}$

$x = x_{max}$

$y = y_{min}$

$y = 0$

Motion Sensor

Figure 5

5. Click the *Start* button in the **DataStudio** program and then the *Stop* button after about 10 seconds.

6. Your graph will be similar to Figure 6. You may adjust the vertical scale (position scale) to get the best display as you did in step 7 of Exercise 1. Read the values y_{min} and y_{max} using the *Smart tool* of the **DataStudio.** To do this, click on the *Smart tool* button 🔳 and then use the arrow keys of the keyboard to move the tool to the correct positions. The numbers in the parenthesis are the coordinates of the point.
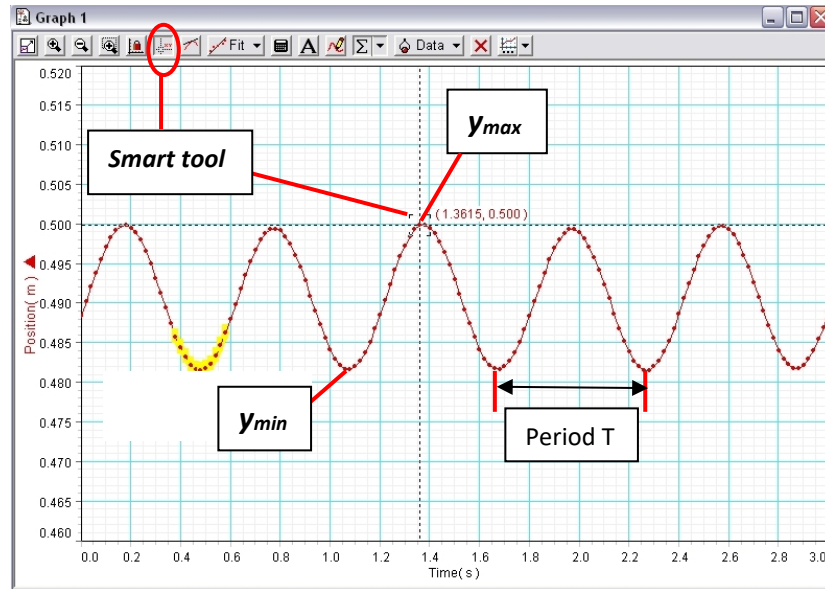

Figure 6

7. Use the value of y_0 obtained from Exercise 1 to calculate:

$$x_{min} = y_0 - y_{max} \quad \text{and} \quad x_{max} = y_0 - y_{min}$$

Record the values in another Table in your report.

Y_{max} (m)	Y_{min} (m)	$x_{max} = y_0 - y_{min}$ (m)	$x_{min} = y_0 - y_{max}$ (m)

Since the spring-mass system is momentarily at rest at both y_{min} and y_{max} the kinetic energy of the system is zero at both of the turning points. Therefore, conservation of energy requires that the increase in gravitational potential energy in going from y_{min} to y_{max} is equal to the decrease in spring potential energy between those two points, so that the change in total mechanical energy is zero. That is,

$$m\,g\,(y_{max} - y_{min}) = \frac{1}{2}k\,(x_{max}^2 - x_{min}^2)$$

8. Use the value of k obtained from Exercise 1 to calculate the left hand side (LHS) and the right hand side (RHS) of the last equation.
9. Calculate the percent difference between (LHS) and (RHS) of the equation.
10. Write a conclusion based on the result of the previous point.

Exercise 3 – The effect of air resistance (Demo, to be done by the instructor)

When a non-conservative force, such as friction or air resistance, is present within the system the total mechanical energy of the isolated system will decrease with time.

In this exercise, you will see the effect of air resistance on the oscillatory motion of the spring-mass system.
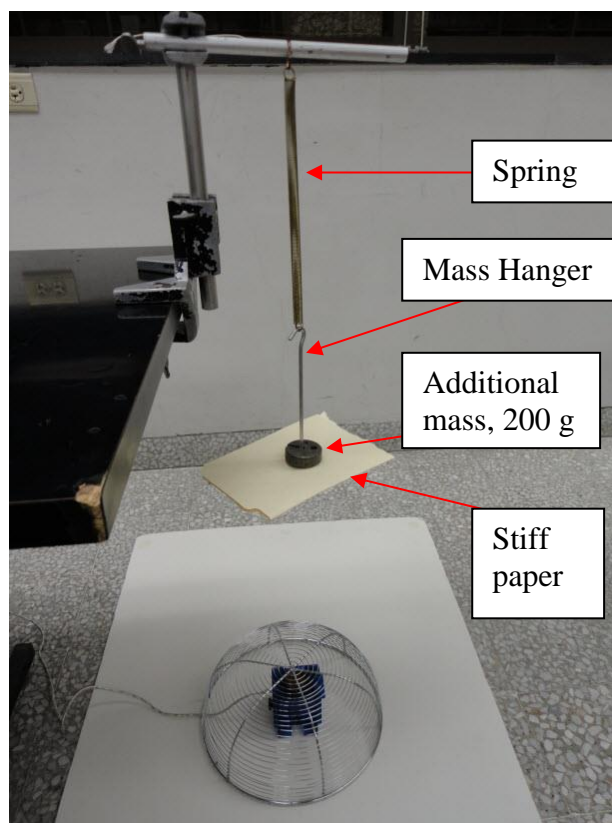
1. Attach a paper to the mass hanger as shown in Figure 7.

Figure 7

2. Click on the **Settings** button ( ) and choose the tab **Axis Settings** in it. Change the x-axis (**time**) scale as 0 to 40 s.

3. Pull the mass slightly down and let it go. This will make the spring-mass system to oscillate. Make sure this oscillation is only along the vertical line.

4. Use **DataStudio** to record the position of the mass as a function of time, as you did in Exercise 2. You should get a graph similar to Figure 8. Note that the y-axis scale is also adjusted to show the best display.
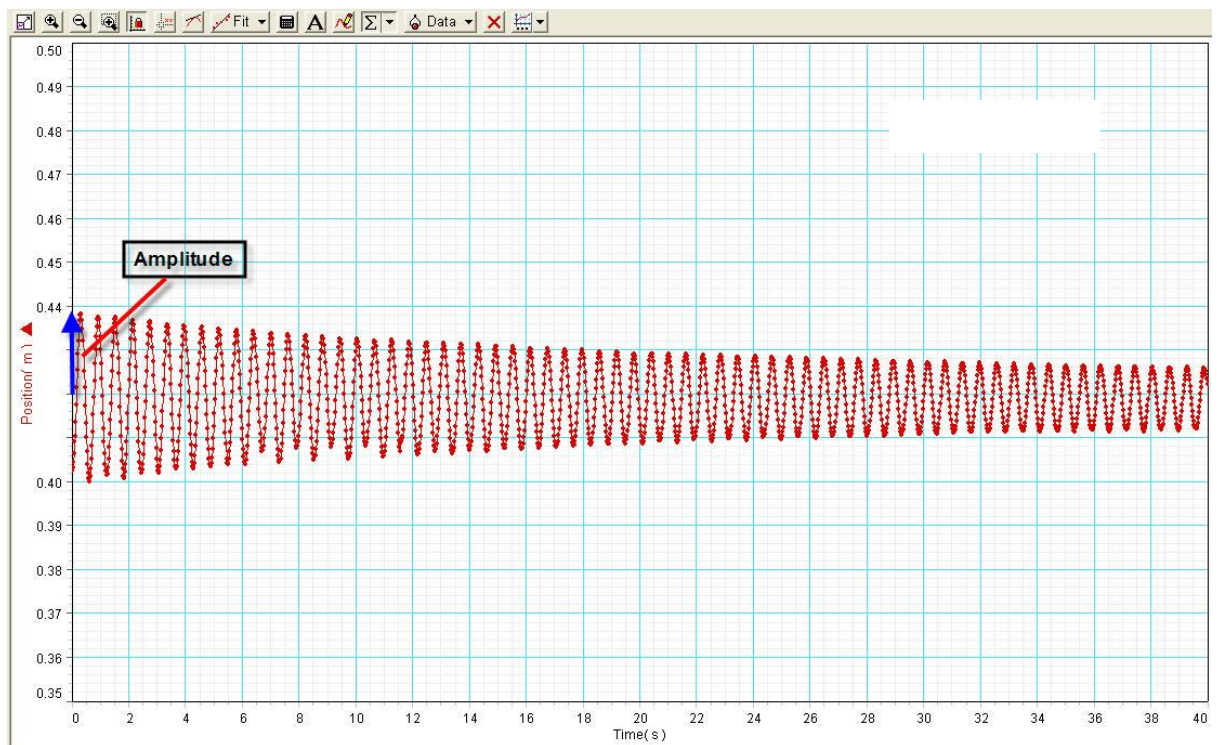
Figure 8

Notice the damping (decreasing amplitude with time) of oscillations of the spring-mass system. This is caused by the air resistance on the paper. Note that the effect of air resistance in Exercise 2, without paper, was negligible. The mechanical energy at y = y_{max} is given by

$$E = m\,g\,y_{max} + \frac{1}{2}k\,x_{min}^2 = m\,g\,y_{max} + \frac{1}{2}k\,(y_0 - y_{max})^2$$

Since m, g, k, and y_0 are constants, y_{max} has to be constant for E to be constant. Therefore, the damped oscillations indicate that the mechanical energy of the system is not conserved. This is due to the presence of the non-conservative force, the air resistance.