Vector Addition

Objectives

To practice adding vectors algebraically.

Introduction

A vector quantity has magnitude and direction and it is denoted by a small arrow on its symbol. For example, force $\vec{F}$ is a vector quantity. The symbol without an arrow F indicates the magnitude. The vector $-\vec{F}$ is a vector with the same magnitude as F but in the opposite direction.

Graphically, a vector is drawn as an arrow indicating its direction. The length of the arrow is proportional to the magnitude of the vector. Two vectors are equal if they are parallel and have same length. So you will not change a vector if you move it in your plot as long as you do not change its direction or its length as shown in Figure 1.

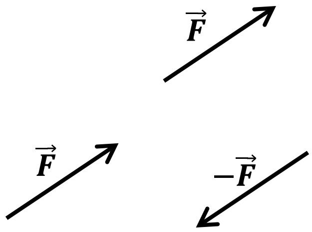


Figure 1

A vector can be specified either by its magnitude and direction or by its components.

$$\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath}$$

where $\hat{\imath}$ is a unit vector along the x-axis and $\hat{\jmath}$ is a unit vector along the y-axis. They have a magnitude of one.

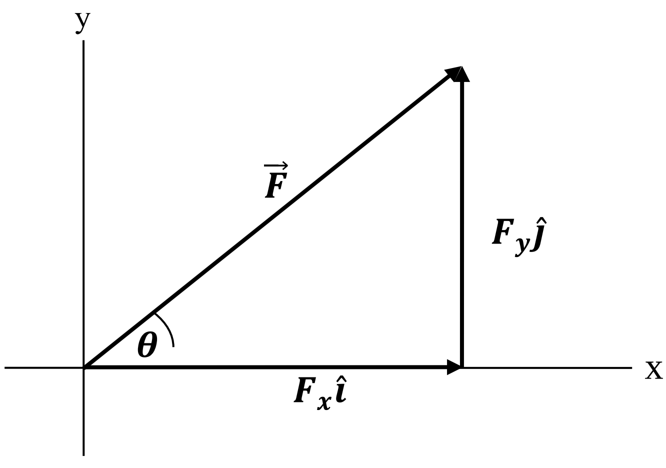


Figure 2

The magnitude of a vector *F* is related to its components through the Pythagorean Theorem.

$$\boldsymbol{F} = \sqrt{\boldsymbol{F}_x^2 + \boldsymbol{F}_y^2}$$

The angle a vector makes with the x-axis θ is related to its components through

$$\boldsymbol{\theta} = \tan^{-1}\frac{\boldsymbol{F}_y}{\boldsymbol{F}_x}$$

The components of a vector is related to its magnitude and the angle with respect to the x-axis θ as follows

$$\boldsymbol{F}_x = \boldsymbol{F}\cos\boldsymbol{\theta}$$
$$\boldsymbol{F}_y = \boldsymbol{F}\sin\boldsymbol{\theta}$$

When vector $\vec{\boldsymbol{A}}$ is added to vector $\vec{\boldsymbol{B}}$, the components of the resultant vector $\vec{\boldsymbol{C}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}$ can be obtained as follows: The x-component of $\vec{\boldsymbol{C}}$ is the sum of the x-components of $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$. The y-component of $\vec{\boldsymbol{C}}$ is the sum of the y-components of $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$.

$$\boldsymbol{C}_x = \boldsymbol{A}_x + \boldsymbol{B}_x$$

$$\boldsymbol{C}_y = \boldsymbol{A}_y + \boldsymbol{B}_y$$

In this lab, you will use tension forces in strings to study some properties of vectors. The tension force is produced by hanging masses from the string.

Figure 3 shows a force table that you will use to study vector addition.

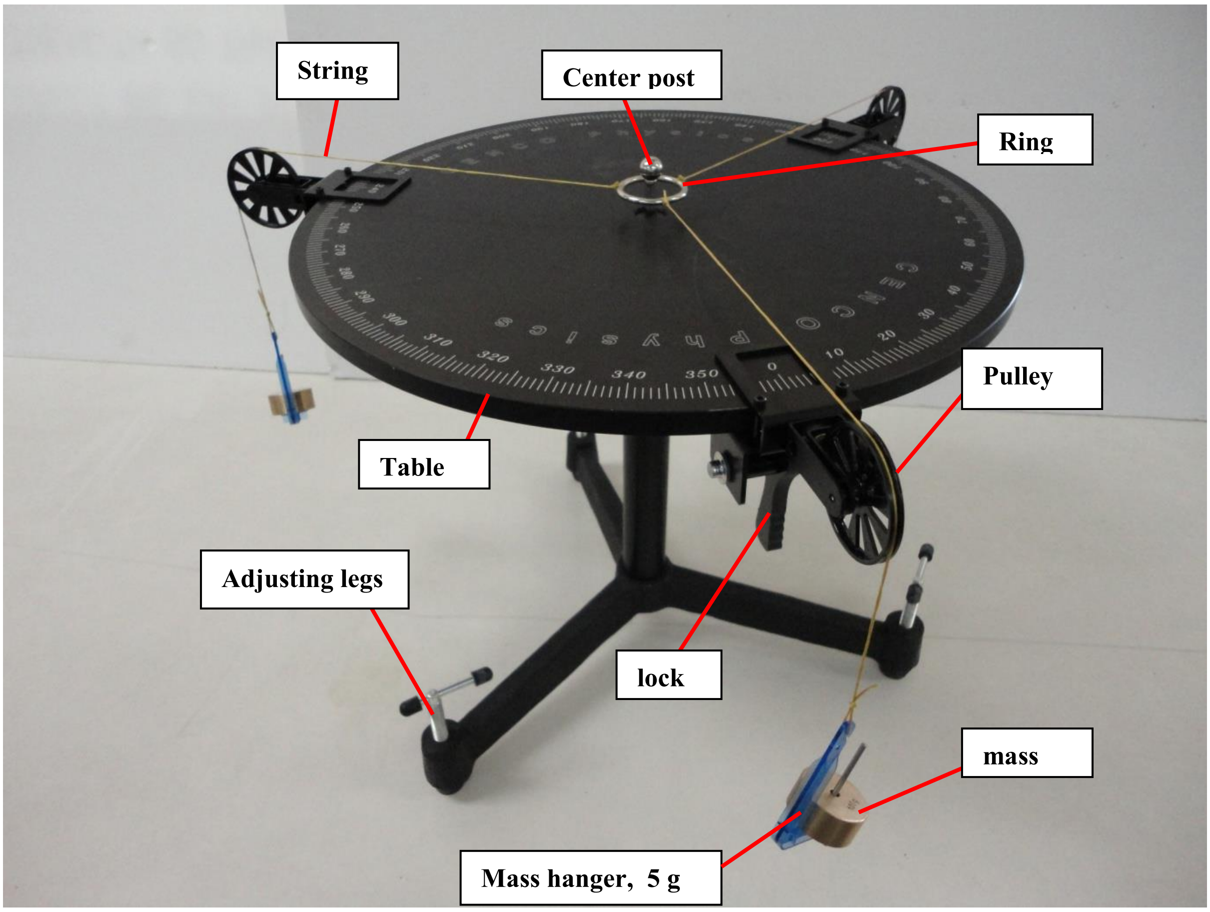


Figure 3: Force Table

Exercise 1 – A vector can be replaced by its components.

In this exercise you will show that the components of a vector have the same effect as the vector itself. That is you can replace a vector by its components.

1. Attach two pulleys to the force table and set one of them at 35° and the other at 35°+180°. If the pulleys are already attached to the table, you have to release the **lock** (see the figure) by pulling it towards you before trying to move the pulley.

2. Place the ring with two strings on the force table, and pass each string over a pulley.

3. **Attach a mass hanger, which has a mass of 5 g, to each string. Add 145 g to each mass hanger so that the total mass pulling each string is 150.0 g.**

4. Move the ring to the center of the table. The ring should stay at rest at the center since the sum of all forces on the ring is zero. The forces on the ring are produced by the pull of the masses hung from the string. As shown in Figure 4, there are two forces $\vec{F}_1$ and $\vec{F}_2$ on the ring.

$$\vec{F}_1 + \vec{F}_2 = \mathbf{0} \qquad \text{or} \qquad \vec{F}_1 = -\vec{F}_2.$$

This means that $\vec{F}_1$ and $\vec{F}_2$ are equal in magnitude and opposite in direction. That is $F_1 = F_2$ and the angle between them is 180^0.

<u>For the rest of this lab, do not change the angle or the mass for $\vec{F}_2$.</u>

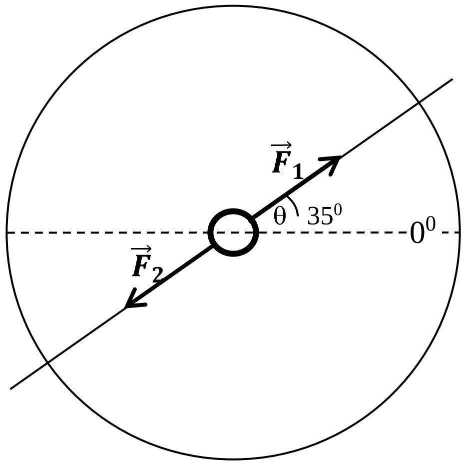


Figure 4

You would like to replace $\vec{F}_1$ by its components $F_{1x}\hat{\imath}$ and $F_{1y}\hat{\jmath}$ and check that these components have the same effect as $\vec{F}_1$. That is $F_{1x}\hat{\imath}$ and $F_{1y}\hat{\jmath}$ should balance $\vec{F}_2$ on the force table as shown in Figure 5.

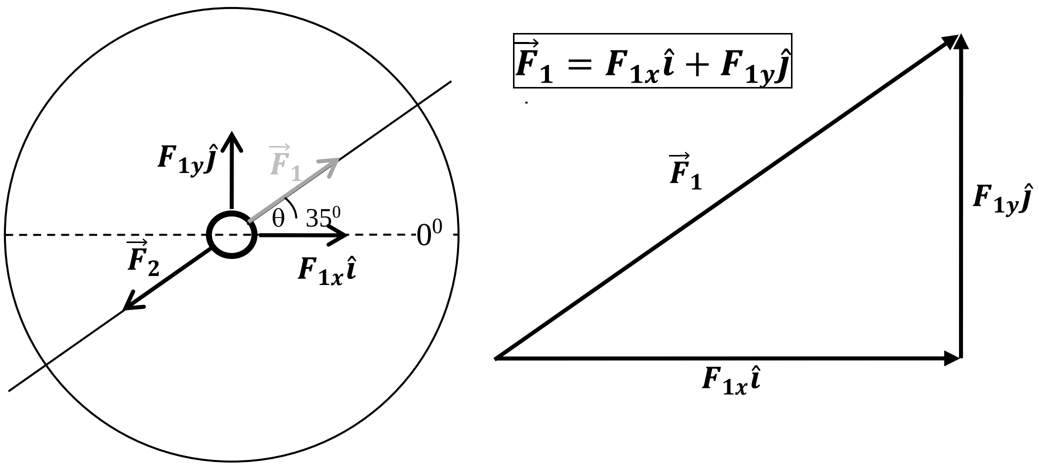


Figure 5

Before doing this, you need to calculate these components.

Figure 6

1. Calculate $F_{1x} = F_1 \cos\theta$

2. Calculate $F_{1y} = F_1 \sin\theta$

3. **Write the calculated values of F_{1x}, F_{1y} in your report. Round off your calculated values to one decimal place.**

To check that $\vec{F}_1$ can be replaced by $F_{1x}\hat{\imath}$ and $F_{1y}\hat{\jmath}$ on the force table,

4. Move the pulley, originally used for $\vec{F}_1$, to 0^0. Now you will use this pulley for $F_{1x}\hat{\imath}$. Change the value of mass hanging from this pulley to that you calculated for x-component, approximated to the nearest integer. This is because you are provided with minimum of only 1 gram masses; as such you will not be able to match the calculated value exactly to the decimal points. But this does not matter as the small friction in the pulley will be able to compensate for this.

5. For $F_{1y}\hat{\jmath}$, attach an extra pulley at 90^0, and hang a mass that corresponds to the calculated y-component, approximated to the nearest integer.

6. This should make the center post at the center of the ring. If not, then pull the strings gently to make the ring centered. You need to pull the strings to overcome small friction in the pulley. If the ring is still not centered, then you need to add or remove a few grams from the mass hangers of $F_{1x}\hat{\imath}$ and $F_{1y}\hat{\jmath}$. The reason for adding or removing small masses is that the value written on the masses may not be accurate.

7. **Record the masses used for $F_{1x}\hat{\imath}$ and $F_{1y}\hat{\jmath}$ in your report. These are your experimental values.**

8. One way to compare experimental values with calculated values is to calculate the percent difference which is calculated as follows:

$$\textbf{Percent difference} = \left|\frac{\textbf{Experimental value} - \textbf{Calculated value}}{\textbf{Calculated value}}\right| \times \textbf{100}$$

9. **Record the percent difference in F_{1x} and F_{1y} in your report.**

Exercise 2 – Adding two vectors.

Suppose $\vec{F}_1$ of Exercise 1 is the vector sum of $\vec{F}_3$ and $\vec{F}_4$.

$$\vec{F}_1 = \vec{F}_3 + \vec{F}_4$$

You will show that $\vec{F}_3$ and $\vec{F}_4$ have the same effect as $\vec{F}_1$ and they can replace it.

<u>Let $\vec{F}_3$ has a magnitude of 80 and makes 20^0 with x-axis. What is the magnitude and the direction of $\vec{F}_4$?</u>

Calculate the magnitude and the direction of $\vec{F}_4$, then check that $\vec{F}_1$ can be replaced by $\vec{F}_3$ and $\vec{F}_4$ on the force table.

The previous equation can be rewritten as

$$\vec{F}_4 = \vec{F}_1 - \vec{F}_3$$

This means that the x-components of these forces satisfy

$$F_{4x} = F_{1x} - F_{3x}$$

And the y-components satisfy

$$F_{4y} = F_{1y} - F_{3y}$$

Figure 7

1. Calculate the x- and y-components of $\vec{F}_1$ and $\vec{F}_3$ as you did in Exercise 1.

Next you need to find the x- and y-components of $\vec{F}_4$ from the above equations

2. The Pythagorean Theorem relates the magnitude of $\vec{F}_4$ to its components.

$$F_4 = \sqrt{F_{4x}^2 + F_{4y}^2}$$

3. To find the angle $\vec{F}_4$ makes with the x-axis, $\theta_4 = \tan^{-1}\frac{F_{4y}}{F_{4x}}$.

4. **<u>In your report, write the calculated values of F_4 and θ_4.</u>** Round off your calculated values to one decimal place.

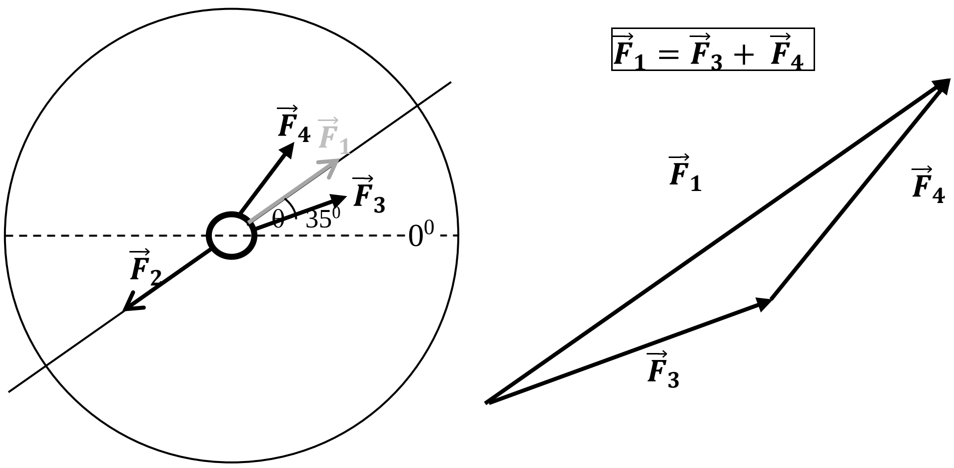


Figure 8

5. Check on the force table that $\vec{F}_1$ can be replaced by $\vec{F}_3$ and $\vec{F}_4$, as you have done with the components of $\vec{F}_1$ in Exercise 1. To do this, move the pulley at 0^0 to 20^0 and change the mass hanging from this pulley to 80 g. Move the pulley at 90^0 to the angle you have calculated for $\vec{F}_4$ and hang a mass that corresponds to $\vec{F}_4$. See Figure 8.

6. This should make the center post at the center of the ring. If not, then pull the strings gently to make the ring centered. You need to pull the strings to overcome small friction in the pulley. If the ring is still not centered, then you need to add or remove a few grams from the mass hangers of $\vec{F}_3$ and $\vec{F}_4$.

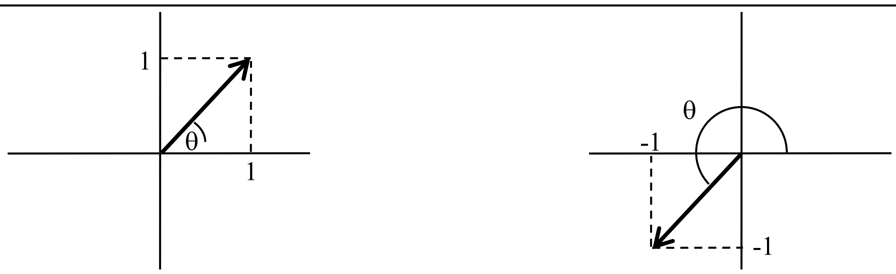
7. **Record in your report the mass used for $\vec{F}_4$ as the measured value, and the percent difference between the measured and calculated values.**

Exercise 3 – Finding the correct angle from the vector components.

In this exercise you will learn how to find the angle from vector components correctly.

Whenever you calculate the angle of a vector from its components using $\tan^{-1}$ function, you need to check the correct quadrant. An example is explained in Figure 9.

In general, if both components are negative, then the angle is in the third quadrant. If the x-component is negative and the y-component is positive then the angle is in the second quadrant. For these two cases, the correct angle is $180^0 + \tan^{-1}\frac{F_y}{F_x}$.

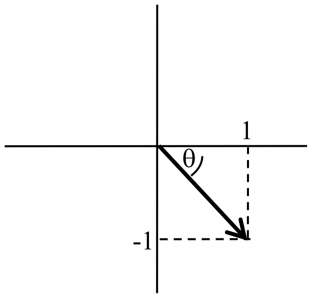


$\theta = \tan^{-1}\frac{1}{1} = 45^0$ Correct!

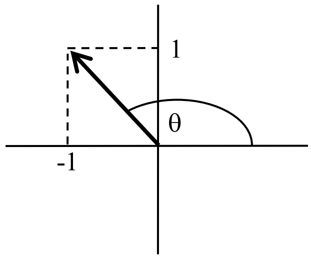
$\theta = \tan^{-1}\frac{-1}{-1} = 45^0$ Wrong!

You need to add 180^0 to get the correct angle!

$\theta = 45^0 + 180^0 = 225^0$ Correct!



$\boldsymbol{\theta} = \tan^{-1}\frac{-1}{1} = -45^0$ Correct!



$\boldsymbol{\theta} = \tan^{-1}\frac{1}{-1} = -45^0$ Wrong!

You need to add 180^0 to get the correct angle!

$\boldsymbol{\theta} = -45^0 + 180^0 = 135^0$ Correct!

Figure 9

You will repeat Exercise 2 with different forces. Suppose $\vec{F}_1$ of Exercise 1 is the vector sum of $\vec{F}_5$ and $\vec{F}_6$.

$$\vec{F}_1 = \vec{F}_5 + \vec{F}_6$$

You will show that $\vec{F}_5$ and $\vec{F}_6$ have the same effect as $\vec{F}_1$ and they can replace it.

Let $\vec{F}_5$ be along the x-axis (θ = 0) with a magnitude of 150. Find the magnitude and the direction of $\vec{F}_6$. **Record your findings in your report.** Round off your calculated values to one decimal place.