# Significant Figures

## Objectives

To understand the concept of significant figures and to learn how it relates to measurements in physics lab.

## Uncertainty

All measurements are approximations; no measuring device can give perfect measurements without experimental uncertainty. For example, if a ruler has marks every centimeter, and a calculator falls between the fourteenth and fifteenth marks as shown in Figure 1, you can be certain that the calculator is longer than 14 cm and less than 15 cm. To get a better idea of how long the calculator actually is, you will have to read between the scale division marks. This is done by estimating the measurement to the nearest one tenth of the space between scale divisions. This estimation introduces the uncertainty in the measurements.



**Figure 1**

You may now estimate the length of the calculator as 14.3 or 14.4 or even 14.5 cm. The best you can do probably is to record it as 14.4 ± 0.1 cm. It means that your measurement is between

14.3 and 14.5 cm. The value 0.1 cm is said to be the uncertainty of this measurement from this ruler. Note that the doubt is in the last digit recorded.

On the other hand, if the ruler has marks every millimeter, and the calculator falls between the 144th and 145th marks as shown in Figure 2, you may record its length as 14.45 ± 0.05 cm. For very small scale divisions judged with our eyes, it is reasonable to take half of the smallest division as the uncertainty of the measurement. Note that the uncertainty of the measurements is now 0.05 cm with the more sensitive ruler. Note also that in the previous case, where the scale division was as large as 1 cm, you were able to estimate to the nearest one tenth of the space between scale divisions whereas with 1 mm scale division you were able to estimate only to the half of the space between scale divisions.
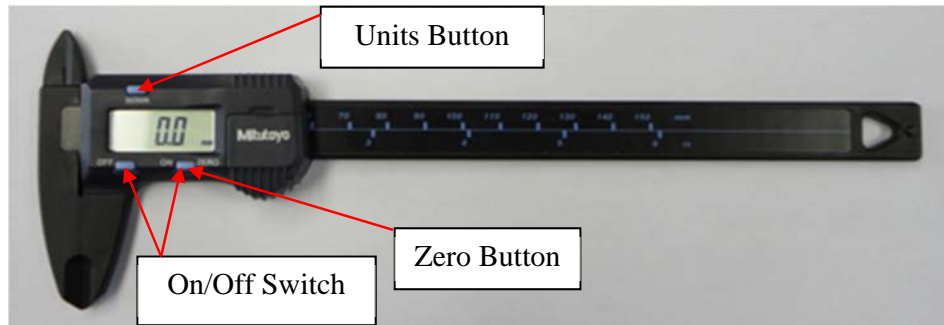


**Figure 2**

**Exercise 1     Measurement with Ruler**

Measure the diameter and height of the given disk using a ruler. Estimate the uncertainty in your measurements. <u>Record your results in your report</u>.

**Exercise 2      Measurements with Digital Caliper**

In this exercise you will measure the diameter and height of the disk using a digital caliper.

1. Make sure to zero the caliper when it is completely closed, and to use mm scale. See Figure 3.
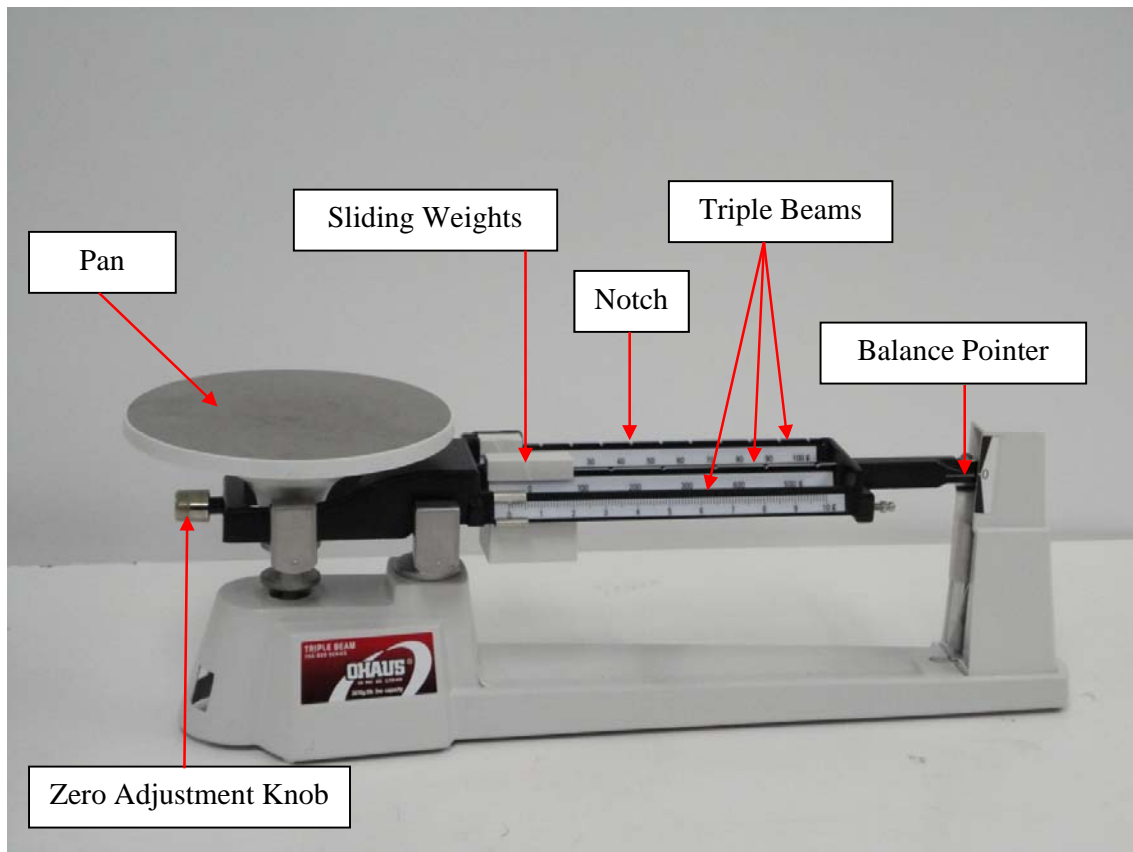


**Figure 3**

2. Measure the diameter of the disk as shown in Figure 4.

3. Measure the height of the disk using the caliper.

4. Estimate the uncertainty in your measurements. It is the standard practice to report the smallest measurement from a digital device as its reading uncertainty.

5. Record your results in your report.



**Figure 4**

**Exercise 3     Measurement with Triple-beam Balance**

In this exercise you will measure the mass of the disk using a triple-beam balance, shown in Figure 5.



**Figure 5**

1.  Make sure the balance pointer indicates zero when the pan is empty and all three sliding weights are at their leftmost positions. If not, use the zero adjustment knob to zero the balance first.

2.  Measure the mass of the disk by placing it on the pan. Make certain that the sliding weights sit firmly in the notches.

3.  Estimate the uncertainty in your measurement. Note that as with a ruler, it is possible to read the front scale to the nearest half of the smallest division.

4.  Measure the mass of the same disk using two other triple-beam balances. Are the three measurements same? If not, suggest a reason.

5.  Record your results in your report.

**Exercise 4      Measurements with Stop Watch**

In this exercise you will measure the time taken for an oscillation, known as the period T, of the simple pendulum using a stop watch, shown in Figure 6. One oscillation is the motion taken for the bob to go from position A (initial position) through C (center position) to B (the other extreme position) and back to A.
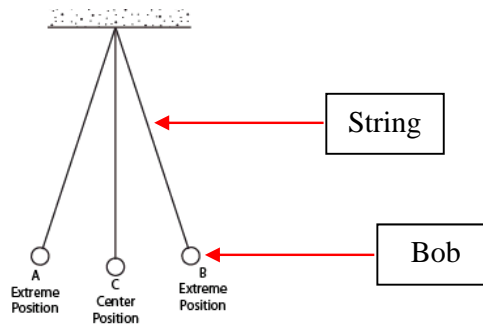


**Figure 6**

1. Measure the period T of the simple pendulum.

2. Estimate the reading uncertainty in your measurement from the stop watch, as you did for the caliper in Exercise 2 and <u>record it in the report</u>.

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3. Repeat the measurement of T four more times.

4. In your report template, **Double Click** on the Excel table and <u>record your 5 values of T in one column</u>. Note that the menu on the top changes to Excel when you double click on the table.

5. Using Excel, determine the average $T_{av}$ of the five $T$ values. To do this, click on an empty cell and click on *Formulas* tab on the menu bar and then click on *Insert Function* button. This will open the Insert Function window. Select *All* in category and click on *AVERAGE* in function. Then click *OK* button. This will open the Function Arguments window. Type the starting and ending cell numbers separated by a semicolon in the Number 1 slot. For example, if you want to find the average of numbers in the cells B2 to B6 then you type B2:B6. Click *OK* button. This will enter the average value in the cell you have chosen. See Figure 7.
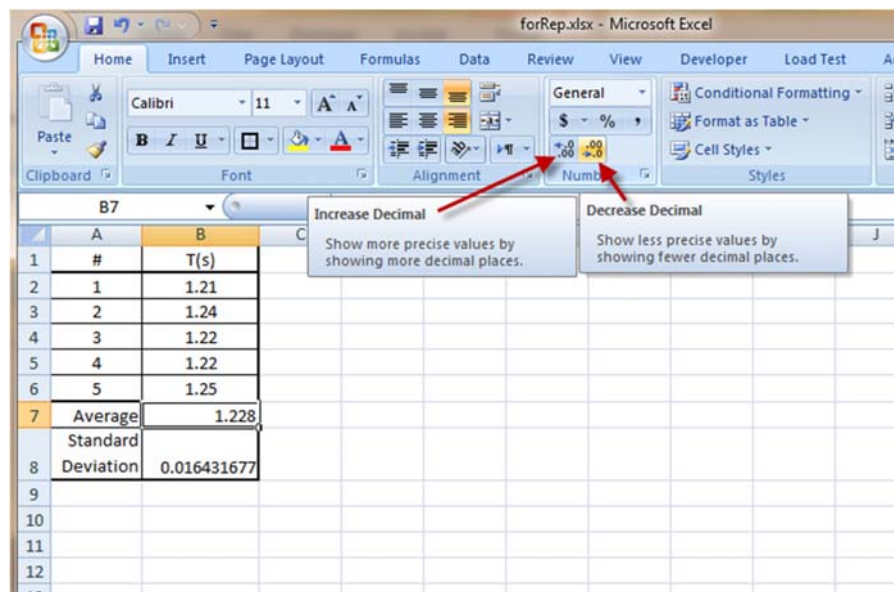


**Figure 7**

6. Following the same procedure described in step 5, determine the standard deviation $\sigma_T$ of the five $T$ values. Here, you will choose the function *STDEV* instead of *AVERAGE*. The standard deviation is a measure of random uncertainty or "error" in the measurement of $T$ due to your response time. Therefore, the measurement $T$ is between the values $(T_{av} - \sigma_T)$ and $(T_{av} + \sigma_T)$, and is recorded as $T_{av} \pm \sigma_T$. This estimate of uncertainty is more likely to be greater than the reading uncertainty. When we have more than one contribution to the uncertainty, the final uncertainty that get reported is the largest one.

7. You should report the average value to only the correct number of significant figures. Use the Decrease/Increase Buttons to change the number of decimal places to be shown to give the right number of significant figures. Standard deviation which is a measure of uncertainty in the measurement should be rounded to have just one significant figure

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only. See Figure 7 for example: for the T values shown in the figure you should be reporting T= 1.23 ± 0.02 (s). Note that the random uncertainty calculated using standard deviation is rounded to one significant figure and found to be greater than the instrumental uncertainty of 0.01 s of the stop watch.

8.  Record the average value of T with its uncertainty in your report. You should use the larger of the two uncertainties estimated.


## Decimal places and Significant figures

The number of decimal places a measurement has is quite straight forward to determine. However, it will differ depending on the units it is being reported. The measurement 14.45 ± 0.05 cm has 2 decimal places but the same measurement in millimeters (144.5 ± 0.5 mm) has only 1 decimal place. Note that the measurement and its uncertainty should have the same number of decimal places.

Another example: $5.34 \times 10^4 = 53400$ has no decimal places
$5.34 \times 10^{-2} = 0.0534$ has 4 decimal places

The number of significant figures in a measurement is simply the number of digits that are known with some degree of reliability. The result 14.4 cm is said to have 3 significant figures and 14.45 cm is said to have 4 significant figures. The precision of an experimental result is implied by the number of digits recorded in the result. Remember when you measured the length of the calculator as 14.4 cm the uncertainty was 0.1 cm whereas when you measured it with more precise ruler as 14.45 cm the uncertainty was 0.05 cm.

Note that the number of significant figures a measurement has does not change when you change the units. Both 14.45 cm and 144.5 mm have 4 significant figures.

Also the number of significant figures should not change when you report in scientific notation. If you are asked to record 133.0 mm in scientific notation, you should write $1.330 \times 10^2$ mm; it is wrong to write it as $1.33 \times 10^2$ mm.

***Rules for deciding the number of significant figures in a measured quantity:***
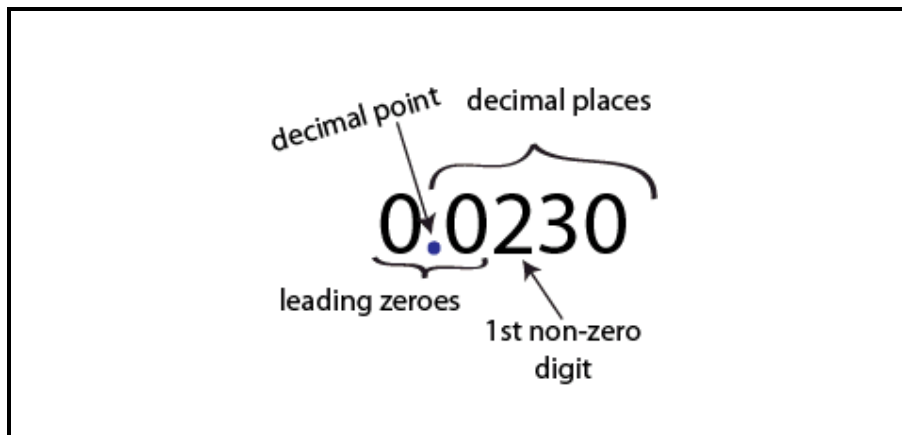
All digits except the leading zeros to the left of the first nonzero digit are significant.

For example,   1002 kg has 4 significant figures.
              3.07 s has 3 significant figures.
              0.001 m has only 1 significant figure.
              0.012 kg has 2 significant figures.
              0.0230 m has 3 significant figures.
              0.20 kg has 2 significant figures.

Note that the leading zeros to the left of the first nonzero digit merely indicate the position of the decimal point.

### Exception to the rule:

When a number ends in zeros that are *not* to the right of a decimal point, the zeros are not necessarily significant.
190 km may have 2 or 3 significant figures.
50,600 s may have 3, 4, or 5 significant figures.

The ambiguity can be avoided by placing a decimal point or by the use of scientific notation. For example, depending on whether the number of significant figures is 3, 4, or 5, we would write 50,600 s as:
$5.06 \times 10^4$ s (3 significant figures)
$5.060 \times 10^4$ s (4 significant figures)
$5.0600 \times 10^4$ s (5 significant figures)

190. has 3 significant figures

### Exact number

Some numbers are exact because they are known with complete certainty. They can be considered to have an infinite number of significant figures. Exact numbers are often found as conversion factors or as counts of objects.
For example,   exactly 10 mm is 1 cm
              Exactly 16 students in a class

Another example: You measured the diameter of a disk to be 23.1 cm and want to determine its radius. Since radius is exactly one half of diameter, $(radius) = \frac{23.1}{2}$ cm . The number **2** in the denominator is obviously an exactly known whole number with no uncertainty in it at all.

**Exercise 5**

Copy the following table in your report and complete it:

| Measurement | Decimal Places | Significant Figures |
|---|---|---|
| 13 | 0 | 2 |
| 13.20 | | |
| 3.0800 | | |
| 0.00418 | | |
| $3.200 \times 10^9$ | | |
| 780.000 | | |
| 0.00800 | | |
| $7.09 \times 10^{-5}$ | | |
| 91 | | |
| 600 | | |
| 250. | | |
| 0.003005 | | |
| 0.0101 | | |

## Rounding off numbers

When insignificant figures are dropped from a number, the last digit kept should be rounded off for the best accuracy. To round off a number to fewer significant digits, you examine the digit following the last digit to be kept in the rounded off number. The digit you are examining is the first digit to be dropped.

1.  If the first digit to be dropped is less than 5 (that is, 1, 2, 3 or 4), drop it along with all the digits to the right of it.
2.  If the first digit to be dropped is more than or equal to 5 (that is, 5, 6, 7, 8 or 9), increase by 1 the number to be rounded, and drop all the digits to the right of it.

For example,   2.1634 when reduced to 4 significant figures becomes 2.163
                        3.608 when reduced to 3 significant figures becomes 3.61

### Exercise 6

Copy the following tables in your report and round the numbers as indicated:

| To four figures: | |
|---|---|
| $2.16347 \times 10^5$ | |
| $4.000574 \times 10^6$ | |
| 3.682417 | |
| 7.2518 | |
| 375.6523 | |
| 21.860051 | |
| | |
| To two figures: | |

| | |
|---|---|
| 3.512 | |
| 158 | |
| 40.523 | |
| $2.751 \times 10^8$ | |
| $3.9814 \times 10^5$ | |
| 22.494 | |

| To one decimal place: | |
|---|---|
| 3.64 | |
| 4.55 | |
| 7.250 | |
| 0.0865 | |
| 0.5182 | |
| 2.473 | |
| 54.7421 | |
| 100.0925 | |
| 0.005 | |
| 79.2588 | |
| 0.9114 | |
| 0.2056 | |

| To nearest 0.01: | |
|---|---|
| 6.675 | |
| 0.4203 | |
| 0.03062 | |
| 4.500 | |
| 2.473 | |
| 7.555 | |

| To the nearest 0.001: | |
|---|---|
| 5.687524 | |
| 39.861214 | |
| 104.97055 | |
| 41.86632 | |
| 0.03765 | |
| 0.0045 | |

| To the nearest whole number: | |
|---|---|
| 56.912 | |
| 3.4125 | |
| 251.7817 | |
| 112.511 | |
| 63.541 | |
| 7.555 | |
| | |

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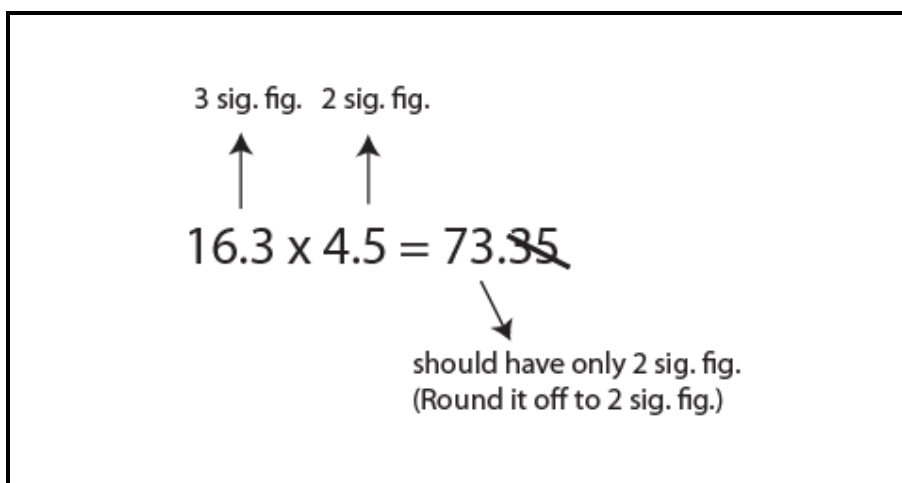| Round off the farthest right digit | |
|---|---|
| 2.473 | |
| 5.396 | |
| 8.235 | |
| 3.05 | |
| 8.25 | |
| 8.65 | |

## Mathematical operations

In carrying out calculations, the general rule is that the accuracy of a calculated result is limited by the least accurate measurement involved in the calculation. A good rule of thumb to use as a guide in determining the number of significant figures in the result is the following:

### 1. For Multiplication and Division

Number of significant figures in the final answer is the same as the number of significant figures in the quantity having the lowest number of significant figures.

For example, suppose that you are asked to measure the area of a rectangular plate using a ruler. Assume that you measured the length = $16.3 \pm 0.1$ cm and the width = $4.5 \pm 0.1$ cm. This means that the length of the plate is between 16.2 and 16.4 cm (3 significant figures), and the width of the plate is between 4.4 and 4.6 cm (2 significant figures). Therefore, the area of the plate is between 16.2 x 4.4 = 71.28 cm$^2$ and 16.4 x 4.6 = 75.44 cm$^2$. Note that the doubt occurs in the second digit. Therefore, the result should be recorded to 2 significant figures. So the area of the plate should be reported as 73 cm$^2$, although your calculator shows 16.3 x 4.5 = 73.35 cm$^2$.



3 sig. fig.   2 sig. fig.

16.3 x 4.5 = 73.35

should have only 2 sig. fig.
(Round it off to 2 sig. fig.)

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### 2. *For Addition and Subtraction*

Number of decimal places in the result should equal the smallest number of decimal places of any term.

For example, suppose you want to find the sum of two measurements $123 \pm 1$ cm and $5.35 \pm 0.05$ cm. It is between $(122+5.30) = 127.30$ cm and $(124+5.40) = 129.40$ cm. Note that the doubt occurs in the third digit. Therefore, the result should be recorded to 3 significant figures. So the sum should be reported as 128, although your calculator shows $(123+5.35) = 128.35$ cm.

```
  123.         0 decimal places

   5.35        2 decimal places
 _____

 128.3̶5̶       0 decimal places
      ↑
   stop here
```

*Warning! Carrying all digits through to the final result before rounding is critical for many mathematical operations. Rounding intermediate results when calculating sums of squares can seriously compromise the accuracy of the result.* ***Therefore, you should apply these rules only at the last step of your calculations.***

Things get a little trickier when you have an operation that mixes addition or subtraction with multiplication or division.

For example, suppose a student determined the magnitude of free-fall acceleration to be 10.2 $m/s^2$, and he calculates the percent difference between his experimental value and the accepted value (9.81 $m/s^2$) using the following formula:

$$\textbf{Percent difference} = \left|\frac{\textbf{Measured value} - \textbf{Accepted value}}{\textbf{Accepted value}}\right| \times \textbf{100}$$

$$= 100 \times (10.2 - 9.81) / 9.81$$

$$= 100 \times (0.39) / 9.81$$

Note that the subtraction gives you 0.39. According to the addition/subtraction rule, this result should be written as 0.4 (to one decimal place) only. **However, you will continue with the calculation without rounding off**. But keep in mind this operation in the bracket should have only one decimal place and therefore it should be considered to be having just one significant figure. Now we have an operation with all multiplication/division only:

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$$= 100 \times (0.39) / 9.81$$

$$= 3.9755 \%$$

100 is an exact number, 0.39 should actually be given to one significant figure, and 9.81 has three significant figures. Therefore the answer should be 4%, with just one significant figure.

**Exercise 7**

In your report template, **Double Click** on the Excel table and perform the following operations as shown in Figure 8. Give your answer with correct number of significant figures. Note that all the numbers in this exercise are measurements. Remember, you can use the Decrease (or Increase) Decimal button in the Number Group of the Home Tab to decrease (or increase) the number of decimal places as you desire.
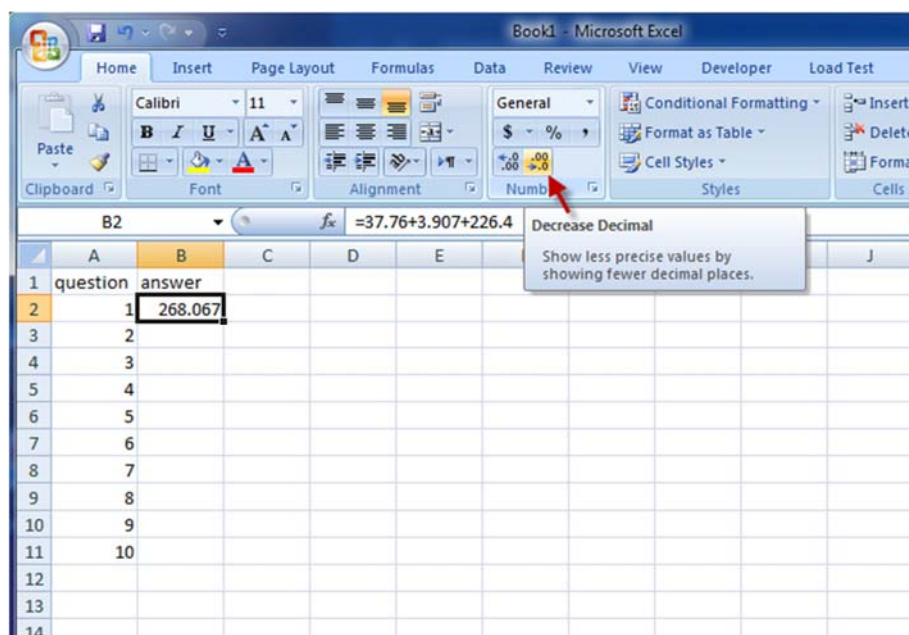


**Figure 8**

1. $37.76 + 3.907 + 226.4 = ?$

2. $319.15 - 32.614 = ?$

3. $104.630 + 27.08362 + 0.61 = ?$

4. $125 - 0.23 + 4.109 = ?$

5. $0.0032 \times 273 = ?$

6. $600.0 / 5.2302 = ?$

7. $(5.5)^3 = ?$

8. $45 \times 3.00 = ?$

9. $0.556 / (40 - 32.5) = ?$

10. $100 \times (10.0 - 9.80) / 9.80 = ?$          (where 100 is exact number)