Q1.

A 6.0 kg particle has the *xy* co-ordinates (–1.20 m, 0.50 m), and a 4.0 kg particle has the *xy* co-ordinates (0.60 m, 0.75 m). Find the *xy* co-ordinates of third particle of mass 2.0 kg if the center of mass of the three-particle system is at the origin.

A) (2.4 m, –3.0 m)
B) (2.4 m, 5.0 m)
C) (3.0 m, –2.4 m)
D) (3.0 m, 2.4 m)
E) (4.4 m, 3.0 m)

Ans:

$$\left(\sum M\right) X_{COM} = m_1 X_1 + m_2 X_2 + m_3 X_3$$

$$(6+4+2)\times 0 = 6(-1.2) + 4(0.6) + 2X_3$$

$$X_3 = \frac{7.2 - 2.4}{2} = 2.4\ m$$

$$\left(\sum M\right) y_{COM} = m_1 y_1 + m_2 y_2 + m_3 y_3$$

$$0 = 6(0.5) + 4(0.75) + 2y_3$$

$$y_3 = \left(\frac{-3-3}{2}\right) = -3m$$

Q2.

A paratrooper whose parachute fails to open lands in snow; he is hurt slightly. Had he landed on bare ground (or hard surface), the stopping time would have been 10 times shorter and the collision lethal (deadly). Which one of the following statements is **True**?

A) The presence of the snow keeps the value of paratrooper’s change in momentum unchanged.
B) The presence of the snow increases the value of paratrooper’s change in momentum.
C) The presence of the snow decreases the value of paratrooper’s change in momentum.
D) The presence of snow increases the impulse stopping the paratrooper.
E) The presence of snow increases the stopping force.

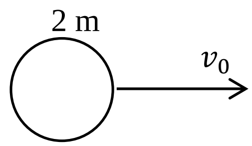
Ans:

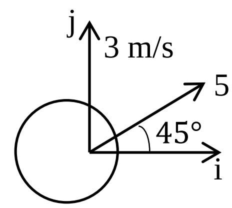
A

Q3.

A 4.0 kg object sliding on a frictionless horizontal surface explodes into two 2.0 kg parts. The first part is travelling with velocity of 3.0 m/s, due north, while the second part travels with the velocity of 5.0 m/s, 45° north of east. What is the original speed of the object?

A) 3.7 m/s
B) 6.3 m/s
C) 7.1 m/s
D) 1.2 m/s
E) 4.5 m/s





Ans:

$$4v_0 = \vec{P}_i$$

$$\vec{P}_f = 2 \times 3\,\hat{j} + 2 \times 5\,sin45°\hat{j} + 2 \times 5 \cos 45°\,\hat{\imath} = 7.1\,\hat{\imath} + 13\,\hat{j}$$

$$\vec{P}_i = \vec{P}_f \Rightarrow \vec{v}_0 = \frac{7.1}{4}\hat{\imath} + \frac{13}{4}\hat{j} = 1.8\hat{\imath} + 3.3\hat{j}$$

$$|v_0| = \sqrt{1.8^2 + 3.3^2} = 3.7\ m/s$$

Q4.

Two spheres, A and B, have different masses. They approach each other (head on) from opposite direction with the initial speed of 3.0 m/s each and collide elastically. After the collision, if sphere A remains at rest what is the final speed of sphere B?

A) 6.0 m/s
B) 4.0 m/s
C) 9.0 m/s
D) 8.0 m/s
E) zero

Ans:

$$V_{Ai} = 3m/s\,;\,v_{Bi} = -3m/s$$

$$V_{Af} = \frac{m_A - m_B}{m_A + m_B} \times 3 + \frac{2m_B}{m_A + m_B} \times -3$$

$$0 = 3m_A - 3m_B - 6m_B \Rightarrow m_A = 3m_B$$

$$P_i = P_f$$

$$m_A v_{Ai} + m_B v_{Bi} = m_B v_{Bf}$$

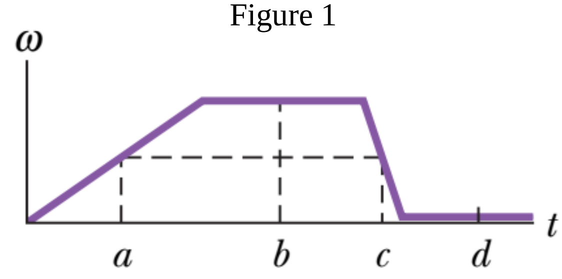
$$3m_B 3 - m_B \times 3 = m_B\, v_{Bf}$$

$$6m_B = m_B v_{Bf} \Rightarrow v_{Bf} = 6\ m/s$$

Q5.

Figure 1 is a graph of the angular velocity versus time for a disk rotating like a merry-go-round. For a point on the disk rim, the magnitudes of maximum tangential acceleration and maximum radial acceleration respectively are at:

Figure 1



A) c and b
B) b and c
C) b and b
D) a and c
E) d and a

Ans:

$$\alpha_t = \frac{d\omega}{dt}, \text{max } at\ c$$

$$\alpha_R = r\omega^2, \text{max } at\ b$$

Q6.

A flywheel makes 40 revolutions as it slows down from an angular speed of 1.50 rad/s to a stop with constant angular deceleration. How much time is required for it to complete the last 20 revolutions?

A) 237 s
B) 126 s
C) 312 s
D) 733 s
E) 431 s

Ans:

$$\theta_0 = 40 \times 2\,\pi \;;\; \omega_0 = 1.5\, rad/s;\; \omega = 0$$

$$\omega^2 = \omega_0^2 + 2\,\alpha\theta_0$$

$$\alpha = -\frac{\omega_0^2}{160\pi}$$

$$\omega_{40} = \omega_{20}^2 + 2\alpha(20 \times 2\pi)$$

$$0 = \omega_{20}^2 + 80\alpha\pi$$

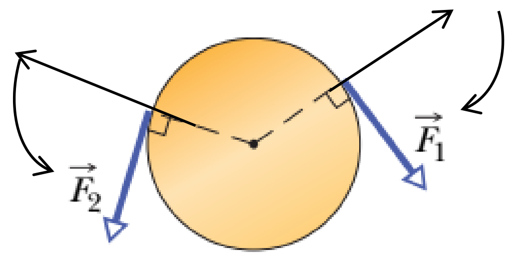
$$\omega_{20}^2 = -80\,\pi\left(-\frac{\omega_0^2}{160\pi}\right) = \frac{\omega_0^2}{2}$$

$$\omega_{40} = \omega_{20} + \alpha t \quad \Rightarrow\ 0 = \frac{\omega_0}{\sqrt{2}} - \frac{\omega_0^2}{160\pi}\,t \quad \Rightarrow t = \frac{160\pi}{\sqrt{2}\,\omega_0}, \quad \Rightarrow t = 237s$$

Q7.

Figure 2 shows a uniform disk that can rotate around its center like a merry-go-round. The disk has a radius of 2.00 cm and a mass of 20.0 grams and is initially at rest. Starting at time t = 0, two forces are to be applied tangentially to the rim as indicated, so that at time t = 1.25 s the disk has an angular velocity of 375 rad/s in the counterclockwise direction. If force $\vec{F}_1$ has a magnitude of 0.100 N, what is the magnitude of $\vec{F}_2$?

Figure 2



A) 0.16 N
B) 0.31 N
C) 0.44 N
D) 0.25 N
E) 0.73 N

Ans:

$$F_2R - F_1R = I\,\alpha$$

$$(F_2 - F_1)R = \frac{1}{2}\,MR^2\,\alpha$$

$$\alpha = \frac{2(F_2 - F_1)}{MR} = \frac{2(F_2 - 0.1)}{20 \times 10^{-3} \times 2 \times 10^{-2}}$$

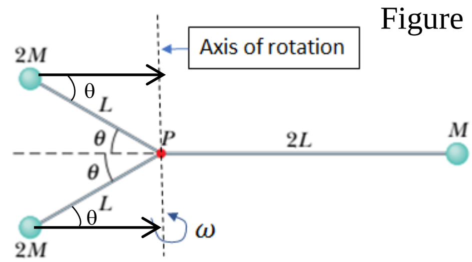
$$F_2 - 0.1 = 20 \times 10^{-5}\,\alpha$$

$$F_2 = 0.1 + 20 \times 10^{-5}\left(\frac{\omega - \omega_0}{t}\right) = 0.1 + 20 \times 10^{-5}\left(\frac{375}{1.25}\right) = 0.16\,N$$

Q8.

The rigid object shown in **Figure 3** consists of three balls and three connecting rods, with M = 1.6 kg, L = 0.6 m and $\boldsymbol{\theta}$ = 30°. The balls may be treated as particles, and the connecting rods have negligible mass. If the object has an angular speed of 1.7 rad/s, determine the rotational kinetic energy of the object about an axis that passes through point P.

Figure 3



A) 5.8 J
B) 3.5 J
C) 1.7 J
D) 8.1 J
E) 2.3 J

Ans:

$$K_R = \frac{1}{2}I\omega^2$$

$$I = M\,(2L)^2 + 2M(L\cos\theta)^2 \times 2$$

$$= 4ML^2(1 + \cos^2\theta) = 4 \times 1.6 \times 0.6^2\left[1 + \cos^2\left(\frac{\pi}{6}\right)\right] = 5.8\text{ J}$$

Q9.

The angular momenta of a particle in four situations are: (1) $l = 3t + 4$; (2) $l = -6t^2$; (3) $l = 2$; (4) $l = -4/t$. Where l = angular momentum and t = time. In which situation is the net torque on the particle for $t > 0$ **negative and increase in magnitude** and **positive and decrease in magnitude** respectively?

A) 2 and 4
B) 4 and 2
C) 1 and 2
D) 2 and 1
E) 1 and 3

Ans:

$$\tau_1 = \frac{dL}{dt} = \frac{d(3t+4)}{dt} = 3$$

$$\tau_3 = \frac{d}{dt}(2) = 0$$

$$\tau_2 = \frac{d}{dt}(-6t^2) = -12\,t\ (Negative\ but\ magnitude\ increase\ with\ time)$$

$$\tau_4 = \frac{d}{dt}\left(-\frac{4}{t}\right) = \frac{4}{t}\ (Positve\ but\ magnitude\ decreases\ with\ time)$$

Q10.

Force $\vec{F} = (2.0N)\hat{i} - (3.0N)\hat{k}$ acts on a pebble (small stone) with position vector $\vec{r} = (0.5m)\hat{i} - (2.0m)\hat{k}$ relative to the origin. Find the magnitude of the resulting torque on the pebble about the point $(2.0\ m, 0, -3.0\ m)$.

A) 2.5 N.m
B) 3.7 N.m
C) 1.2 N.m
D) 6.1 N.m
E) 4.3 N.m

Ans:

$$\tau = \Delta\vec{r} \times \vec{F} = (\vec{r} - \vec{r}_0) \times \vec{F} = \left(0.5\,\hat{\imath} - 2\,\hat{k} - 2\,i + 3\,\hat{k}\right) \times \vec{F}$$

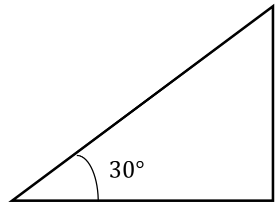
$$\therefore \tau = \left(-1.5\,\hat{\imath} + 1\,\hat{k}\right) \times \left(2\,\hat{\imath} - 3\,\hat{k}\right) = +4.5\left(\hat{\imath} \times \hat{k}\right) + 2\left(\hat{k} \times i\right)$$

$$\tau = -4.5\,\hat{\jmath} + 2\hat{\jmath} = -2.5\,\hat{\jmath}$$

$$|\tau| = 2.5\ \ N \cdot m$$

Q11.

A uniform ball of mass M = 5.0 kg and radius R rolls smoothly from rest down a ramp (inclined plane) at angle θ = 30°. The ball rolls down the vertical height h =1.8 m to reach the bottom of the ramp. What is speed of the ball at the bottom of the ramp?



A) 5.0 m/s
B) 3.0 m/s
C) 7.3 m/s
D) 2.0 m/s
E) 4.2 m/s

Ans:

$$K_0 = 0\ ;\ U_0 = mgh\ ; K = \frac{1}{2}\ I\omega^2 + \frac{1}{2}\ mv^2\ ; U = 0$$

$$\Delta K + \Delta U_g = 0$$

$$\frac{1}{2} I\omega^2 + \frac{1}{2} mv^2 - mgh = 0$$

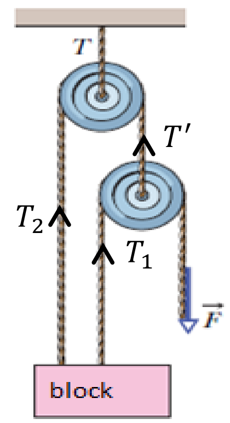
$$\frac{1}{2} \times \frac{2}{5} m\cancel{R^2} \frac{v^2}{\cancel{R^2}} + \frac{1}{2} mv^2 = mgh$$

$$\frac{7}{10} mv^2 = mgh \Rightarrow v = \sqrt{\frac{10gh}{7}} = 5.0\ m/s$$

Q12.

In **Figure 4**, a force $\vec{F}$ keeps a block of weight *mg* in static equilibrium. Considering all pullies as massless and frictionless find the tension *T* on the upper cable.

Figure 4



A) 1.3 *mg*
B) 2.1 *mg*
C) 0.50 *mg*
D) 3.2 *mg*
E) 0.27 *mg*

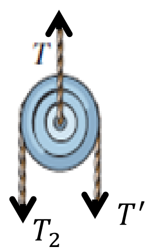
Ans:

$$T_2\, 2R = TR$$

$$T_2 = \frac{T}{2}$$

$$T' \cdot 2R = T \cdot R$$

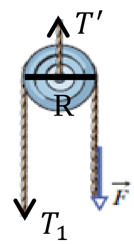
$$T' = \frac{T}{2}$$



$$T'R = T_1 2R$$

$$T' = 2T_1$$

$$T_1 = \frac{T'}{2} = \frac{T}{4}$$

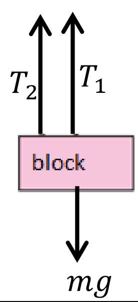


$$T_1 + T_2 = mg$$

$$\frac{T}{4} + \frac{T}{2} = mg$$

$$3T = 4mg$$

$$T = \frac{4}{3}\, mg = 1.3\, mg$$



Q13.

A boy of mass 36.0 kg stands at the center of a merry-go-round like rotating disc of radius 2.00m. The disc has mass of 200 kg and it is rotating with time period of 2.50 s. While it is rotating, the boy walks out to the edge of the disc. What is the rotational period of the disc when the boy gets to the edge?

A) 3.40 s
B) 1.90 s
C) 2.50 s
D) 4.10 s
E) 2.90 s

Ans:

$$I_D\omega_0 = (I_D + I_B)\omega$$

$$\frac{1}{2}MR^2\frac{2\pi}{T_0} = \left(\frac{1}{2}MR^2 + mR^2\right)\frac{2\pi}{T}$$

$$T = \frac{2\left(\frac{M}{2} + m\right)T_0}{M} = \left(1 + \frac{2m}{M}\right)T_0$$

$$\therefore\ T = \left(1 + \frac{2 \times 36}{200}\right) \times 2.5 = 3.40\ s$$

Q14.

In **Figure 5**, a man leans against a vertical wall that has negligible friction. Distance a = 0.91 m and distance L = 2.1 m. His center of mass is at distance d = 0.94 m from the feet-ground contact point. If he is on the verge of sliding, what is the coefficient of static friction between his feet and the ground?

A) 0.22
B) 0.42
C) 0.64
D) 0.12
E) 0.34

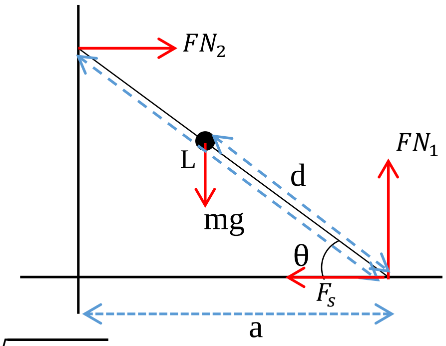
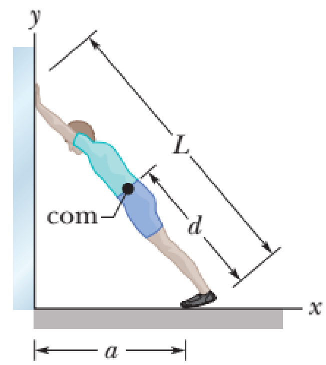


Figure 5



Ans:

Torque equation

$$mgdcos\theta = F_{N2} \cdot \sqrt{L^2 - a^2}$$

$$\frac{mgd}{\sqrt{L^2 - a^2}} \frac{a}{L} = F_{N2}$$

Force equations,

$$F_s = F_{N2}$$

$$F_{N1} = mg$$

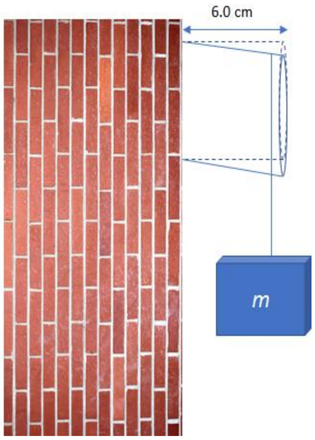
$$F_S = F_{N1} \cdot \mu_s$$

$$\mu_s = \frac{F_S}{F_{N1}} = \frac{F_{N2}}{mg} = \frac{mg\,da}{L\sqrt{L^2 - a^2}} \cdot \frac{1}{mg} = \frac{ad}{L\sqrt{L^2 - a^2}} = 0.22$$

Q15.

A rod 5.0 cm in diameter projects 6.0 cm from a wall as shown in **Figure 6**. An object of mass $m = 1.2\times10^3$ kg is suspended from the end of the rod. The shear modulus of the rod is 3.0×10^{10} N/m^2. Neglecting the rod's mass, what is the vertical deflection of the end of the rod.

Figure 6



A) 1.2×10^{-5} m
B) 2.4×10^{-5} m
C) 3.1×10^{-5} m
D) 5.4×10^{-5} m
E) 6.2×10^{-5} m

Ans:

$$\Delta l = \frac{F/A}{\Delta l/l} = \frac{F}{\pi R^2}\cdot\frac{l}{\Delta l}$$

$$\Delta l = \frac{F}{\pi R^2}\cdot\frac{l}{\sigma} = \frac{1.2\times10^3\times9.8\times6\times10^{-2}}{13.4\times2.5\times10^{-2}\times3\times10^{10}}$$

$$\Delta l = 1.2\times10^{-5}\ m$$