Q1.

The angular position of a point on the rim of a rotating wheel of radius R is given by: θ (t) = 6.0 t + 3.0 t^2 – 2.0 t^3, where θ is in radians and t is in seconds. What is the average angular acceleration for a point at R/2 for the time interval between t = 0 and t = 5 s?

A) – 24 rad/s^2
B) + 24 rad/s^2
C) 0
D) – 12 rad/s^2
E) + 12 rad/s^2

Ans:

$\theta(t) = 6.0\,t + 3.0\,t^2 - 2.0\,t^3 \quad \Rightarrow \quad \omega(t) = 6.0 + 6.0\,t - 6.0\,t^2$

$\omega(0) = 6.0$, $\omega(5) = -114$

$$\Rightarrow \quad \bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{-114-6}{5-0} = -24$$

Q2.

An object of mass m = 15 kg initially at rest explodes into two pieces of masses 10 kg and 5.0 kg. The velocity of the 5.0 kg mass is 4.0 m/s along the positive x-axis. Find the kinetic energy of the 10 kg piece.

A) 20 J
B) 30 J
C) 40 J
D) 50 J
E) 60 J

Ans:

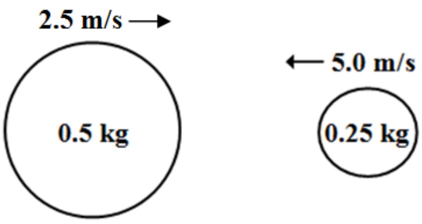
$$P_f = P_i \quad \Rightarrow \quad -10v + 5\times 4 = 0 \quad \Rightarrow \quad v = 2 \text{ m/s}$$

$$\Rightarrow \quad k = \frac{1}{2}mv^2 = \frac{1}{2}\times 10\times 2^2 = 20 \text{ J.}$$

Q3.

Figure 1 shows a 0.5 kg ball moving at 2.5 m/s collides head on with a 0.25 kg ball moving in the opposite direction at 5.0 m/s. Determine the final kinetic energy of the 0.5 kg ball if the collision is perfectly elastic.

Figure 1



A) 1.6 J
B) 2.3 J
C) 6.4 J
D) 11 J
E) 0.11 J

Ans:

$m_1 = 0.5;\ \ v_{1i} = 2.5;\ m_2 = 0.25;\ \ v_{2i} = -5$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} = -2.5$$

$k = 0.5 m_1 v_1 f^2 = 1.5625 = 1.6\ J$

Q4.

A uniform disk starts from rest and rotates, about fixed central axis, with a constant angular acceleration. It reaches an angular velocity of 13.7 rad/s when it has completed 5.00 revolutions. What is the angular velocity when it has completed 9.00 revolutions?

A) 18.4 rad/s
B) 17.2 rad/s
C) 11.2 rad/s
D) 8.20 rad/s
E) 0

Ans:

First calculate the acceleration

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2\Delta\theta} = \frac{(13.7)^2 - 0}{2\times 5(2\pi)} = 2.987 \quad \text{rad/s}^2$$

Second $\omega(9\text{ revolutions}) = \sqrt{\omega_i^2 + 2\alpha\Delta\theta} = \sqrt{0 + 2\times 2.987\times 9\times 2\pi} = 18.38\text{ rad/s}$

Q5.

A uniform disk is rotating with angular velocity ω about a fixed axis perpendicular to its plane and passing through a point on its edge. Find the ratio of its kinetic energy about this axis of rotation to its kinetic energy about a parallel axis passing through its center of mass and rotating with the same angular velocity ω.

A) 3
B) 9
C) $\sqrt{3}$
D) 4
E) 1

Ans:

The ratio is:

$$\frac{K_{edge}}{K_{center}} = \frac{\frac{1}{2}\left(MR^2 + \frac{1}{2}MR^2\right)\omega^2}{\frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2} = 3$$

Q6.

A torque, of 2.0 N·m, is applied to a pulley rotating about fixed central axis. Starting from rest, the angular speed of the pulley after 4.0 s is 120 rev/min. What is the rotational inertia, in kg.m^2, of the pulley?

A) 0.64
B) 0.81
C) 0.22
D) 0.12
E) 1.00

Ans:

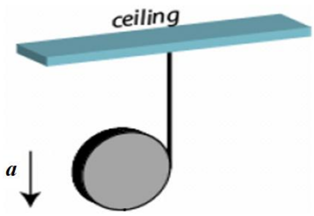
$$\tau = I\alpha \Rightarrow I = \frac{\tau}{\alpha}; \quad \omega = \omega_o + \alpha t \Rightarrow \alpha = \frac{\omega_o}{t} = \frac{120 \times 2\pi / 60}{4} = \pi$$

$$\therefore I = \frac{2}{\pi} = 0.637$$

Q7.

A string (one end attached to the ceiling) is wound around a uniform solid cylinder of mass M = 2.0 kg and radius R = 10 cm (see **Figure 2**). The cylinder starts falling from rest as the string unwinds. The linear acceleration, in m/s^2, of the cylinder is:

Figure 2



A) 6.5
B) 4.3
C) 8.5
D) 1.1
E) 2.2

Ans:

$$ma = mg - T \quad (1),$$

$$\because I_{cm}\alpha = TR; \quad \alpha = \frac{a}{R} \quad \Rightarrow \quad \frac{1}{2}mR^2\left(\frac{a}{R}\right) = TR \quad \Rightarrow \quad \frac{1}{2}ma = T \quad (2)$$

$$\therefore (1) \Rightarrow a = \frac{2}{3}g = 6.53 \text{ m/s}^2$$

Q8.

A hoop rolls without sliding on a horizontal floor. The ratio of its translational kinetic energy to its rotational kinetic energy (about its central axis) is

A) 1
B) 2
C) 3
D) 1/3
E) 1/2

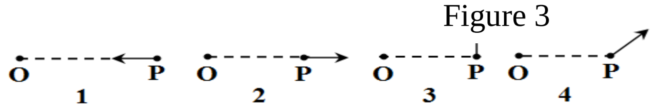
Ans:

$$\frac{K_{edge}}{K_{center}} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}I\omega^2} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}\left(mR^2\right)\left(v/R\right)^2} = 1$$

Q9.

A single force acts on a particle P. Rank each of the orientations of the force shown in **Figure 3** according to the magnitude of the time rate of change of the particle's angular momentum about the point O, least to greatest.

Figure 3



A) 1 and 2 tie, then 4, then 3
B) 1, 2, 3, 4
C) 1 and 2 tie, then 3, then 4
D) 1 and 2 tie, then 3 and 4 tie
E) All are the same

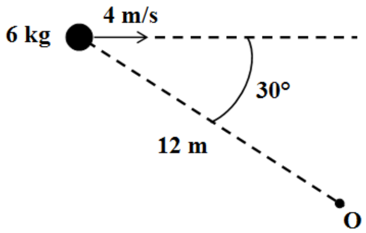
Ans:

A

Q10.

A 6.0 kg particle moves to the right at 4.0 m/s as shown in **Figure 4**. Its angular momentum, in kg.m^2/s, about point O is:

Figure 4



A) 144, into the page
B) 0
C) 249, into the page
D) 144, out of the page
E) 249, out of the page

Ans:

The angle between the tails of the momentum vector and the position vector is 30°;
$L = m\,v\,r\sin 30 = 6(4)(12)\sin 30 = 144$ kg m^2/s into the page

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Q11.

A merry-go-round of radius 2.0 m is rotating about a frictionless pivot. It makes one revolution every 5.0 s. The moment of inertia of the merry-go-round (about an axis through its center) is 500 kg·m^2. A child of mass 25 kg, originally standing at the rim, walks radially in to the exact center. The child can be considered as a point mass. What is the new angular velocity, in rad/sec, of the merry-go-round?

A) 1.5
B) 1.3
C) 2.3
D) 1.9
E) 0.5

Ans:

Apply the conservation of angular momentum (there are no net external torques on the system of merry-go-round and child). Thus we have L = constant = $I_i\,\omega_i$ = $I_f\,\omega_f$ or ω_f = $I_i\,\omega_i / I_f \omega_i$

The initial angular velocity and the initial and final moments of inertia. SinceT = 5 s, so the initial angular velocity is
$\omega_i = 2\pi/T = 1.257$ rad/s

The initial moment-of-inertia is that of the merry-go-round plus that of the child located at the rim:
I_i = 500 Kg·m^2 + mR^2 = 500 Kg·m^2 + (25 kg)(2 m)^2 = 600 kg·m^2

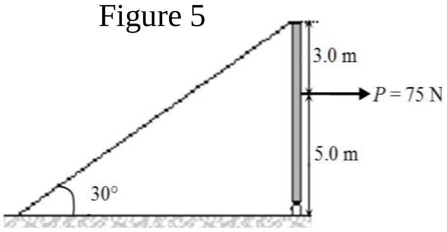
Since the child ends up at the center (r = 0), she/he contributes no rotational inertia in the final situation, so the I_f is just that of the merry-go-round, *i.e.*
I_f = 500 Kg·m^2

Plugging these in gives
ω_f = (600 kg·m^2)(1.257 rad/s)/(500 Kg·m^2)
= 1.51 rad/sec

Q12.

A uniform 100 kg beam is held in a vertical position by a pin at its lower end, a cable at its upper end, and by applying a horizontal force *P* = 75 N as shown in **Figure 5**. Find the tension in the cable.

Figure 5



A) 54 N
B) 99 N
C) 14 N
D) 10 N
E) 76 N

Ans:

Take the torque about the pin

$$\tau_o = T \times 8 \times \cos 30^\circ - 75 \times 5 = 0 \quad \Rightarrow \quad T = \frac{75 \times 5}{8 \times \cos 30^\circ} = 54.13$$

Q13.

A certain wire, hanging from a ceiling, stretches 0.9 cm when outward force with magnitude F is applied to the free end. The same force is applied to a wire of the same material but with three times the diameter and three times the length. The second wire stretches:

A) 0.3 cm
B) 0.1 cm
C) 0.9 cm
D) 2.7 cm
E) 8.1 cm

Ans:

Calculate the ratio:

$$\frac{\Delta L_1}{\Delta L_2} = \frac{F_1 L_1 / A_1 E_1}{F_2 L_2 / A_2 E_2} = \frac{L_1 A_2}{L_2 A_1} = \frac{1L_1 \times \pi (3d/2)^2}{3L_1 \times \pi (d/2)^2} = 3$$

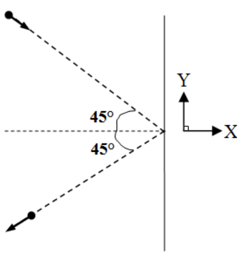
$$\Delta L_2 = \frac{\Delta L_1}{3} = \frac{0.9}{3} = 0.3$$

Q14.

As shown in **Figure 6**, a ball with a mass of 1.0 kg and a speed of 25 m/s hits a vertical wall at an angle of 45° and rebounds with the same speed with the same angle. Find the change in the linear momentum, in $\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}}$, of the ball.

A) $-35\ \hat{i}$
B) $+35\ \hat{i}$
C) $-70\ \hat{i}$
D) $+70\ \hat{i}$
E) $-25\ \hat{j}$

Figure 6



Ans:

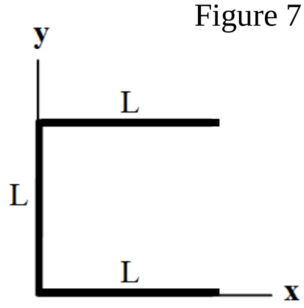
$$\Delta P_y = 0$$

$$\Delta P_x = (-mv - mv)\cos 45^\circ = -2 \times 1 \times 25 \times 0.707 = -35.36$$

Q15.

An object is formed by three identical uniform thin rods, each of length L and mass M, as shown in **Figure 7**. Determine the x and y coordinates, (x, y), of the center of mass of this object.

Figure 7



A) (L/3 , L/2)
B) (0 , L/2)
C) (L , L/2)
D) (L/2 , L)
E) (L/4 , L/4)

Ans:

$$x_{cm} = \frac{M(0) + M(L/2) + M(L/2)}{3M} = L/3$$

$$y_{cm} = \frac{M(L/2) + M(0) + M(L)}{3M} = L/2$$