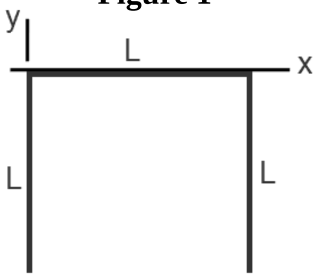
Q1.

Three uniform thin rods, each of length L = 20 cm, form an inverted U shape as shown in **Figure 1**. Each one of the vertical rods has a mass of 20 g and the horizontal rod has a mass of 60 g. What are the **x** and **y** coordinates of the center of mass of the system, respectively?

Figure 1



A) (10 and – 4.0) cm
B) (20 and 1.0) cm
C) (30 and – 6.0) cm
D) (10 and 2.0) cm
E) (4.0 and – 10) cm

Ans:

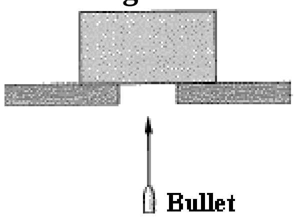
$$X_{com} = \frac{60 \times 10 + 20 \times 20}{20 + 20 + 60} = +10\text{ cm}$$

$$Y_{com} = \frac{20 \times (-10) + 20 \times (-10)}{20 + 20 + 60} = -4.0\text{ cm}$$

Q2.

A 20.0 g bullet moving vertically upward at 1.00×10^3 m/s strikes and passes through the center of mass of a 10.0 kg block initially at rest, as shown in **Figure 2**. To what maximum height does the block rise after the bullet emerges from the block with a speed of 4.00×10^2 m/s vertically upward. Ignore air resistance.

Figure 2



A) 7.35 cm
B) Zero
C) 14.7 cm
D) 1.04 m
E) 2.07 m

Ans:

$$\text{Block final kinetic energy} = \frac{1}{2}V_{Bf}^2$$

v_b is bullet velocity. Then $mv_{bi} = mv_{bf} + MV_{Bf}$

$$V_{Bf} = \frac{m(v_{bi} - v_{bf})}{M}$$

$$= \frac{0.02 \times (10^3 - 4 \times 10^2)}{10} = 1.2\text{ m/s}$$

For block maximum height:

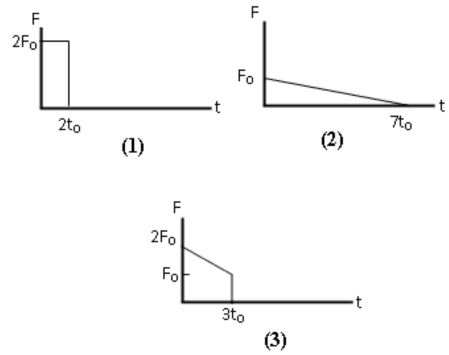
$$\cancel{M}gh = \frac{1}{2}\cancel{M}V_{Bf}^2$$

$$h = \frac{(V_{Bf})^2}{2g} = \frac{(1.2)^2}{2 \times 9.8} = 0.0735 = 7.35\text{ cm}$$

Q3.

Figure 3 shows graphs of force magnitude versus time for a body involved in a collision. Rank the graphs according to the magnitude of the impulse on the body, **smallest** first.

Figure 3



A) 2, 1, 3
B) 1, 2, 3
C) 2, 3, 1
D) 3, 2, 1
E) 1, 3, 2

Ans:

Impulse manitude $=$ Area under the graph

$$\text{Area } 1 = 2t_0 \times 2F_0 = 4F_0t_0$$
$$\text{Area } 2 = \frac{1}{2} \times 7t_0 \times F_0 = \frac{7}{2}F_0t_0$$
$$\text{Area } 3 = 3t_0 \times F_0 = \frac{1}{2} \times 3t_0 \times F_0 = \frac{9}{2} F_0t_0$$

Areas smallest to the largest = 2, 1, 3

Q4.

Block 1 of mass 1.00 kg is moving with an initial speed of 4.00 m/s on a frictionless horizontal surface. It makes an elastic head on collision with a stationary block 2 of mass 12.0 kg. What is the kinetic energy of block 2 after collision?

A) 2.27 J
B) 7.22 J
C) 1.15 J
D) 4.03 J
E) 3.43 J

Ans:

$$K_{2f} = \frac{1}{2}m_2 v_{2f}^2 \;;\; v_{2f} = \frac{2m_1}{m_1 + m_2};\; v_{1i} = \left(\frac{2 \times 1}{1 + 12}\right) \times 4 = \frac{8}{13}\ \text{m/s}$$

$$K_{2f} = \frac{1}{2}m_2 v_{2f}^2 = \frac{1}{2} \times 12 \times \left(\frac{8}{13}\right)^2 = 2.27\ \text{J}$$

Q5.

Two 2.00 kg bodies, A and B, collide. Their velocities before the collision are $\vec{v}_{Ai} = (1.50\,\hat{i} + 3.00\,\hat{j})$ m/s and $\vec{v}_{Bi} = (-1.00\,\hat{i} + 0.500\,\hat{j})$ m/s. After the collision, velocity of A is $\vec{v}_{Af} = (-0.500\,\hat{i} + 2.00\,\hat{j})$ m/s. What is the kinetic energy of body B after the collision?

A) 3.25 J
B) 5.04 J
C) 7.53 J
D) 1.50 J
E) 9.52 J

Ans:

$$\vec{v}_{Bf} = \frac{m_A\vec{v}_{Ai} + m_B v_{Bi} - m_A\vec{v}_{Af}}{m_2} = \frac{m_1(\vec{v}_{Ai} - \vec{v}_{Af}) + m_B v_{Bi}}{m_2}$$

$$= \frac{\cancel{2}(1.50\,\vec{\imath} + 3.00\,\vec{\jmath} + 0.5\,\vec{\imath} - 2.00\,\vec{\jmath}) + \cancel{2}(-1.0\,\vec{\imath} + 0.5\,\vec{\jmath})}{\cancel{2}}$$

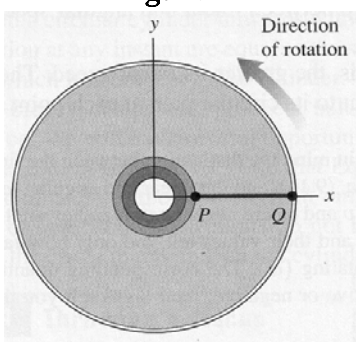
$$\vec{v}_{Bf} = 1.01\ \vec{\imath} + 1.5\,\vec{\jmath}$$

$$K_{Bf} = \frac{1}{2} m_B {v_{Bf}}^2 = \frac{1}{2} \times 2 \times (1.0^2 + 1.5^2) = 3.25\ \text{J}$$

Q6.

As shown in **Figure 4**, a disk rotates in the xy plane about the z-axis passing through the disk center with a constant angular acceleration of – 5.00 rad/s^2. At time t = 0, the disk's angular speed is 27.5 rad/s and a line PQ on the disk's surface coincides with the positive x-axis. What angle does the line PQ make with the positive x-axis at time t = 1.40 s?

Figure 4



A) 2.18 rad
B) 1.93 rad
C) 0.75 rad
D) 3.34 rad
E) 0.47 rad

Ans:

$$\Delta\theta = \omega_1 t + \frac{1}{2}\alpha t^2 = 27.5 \times 1.4 + \frac{1}{2} \times (-5) \times (1.4)^2$$

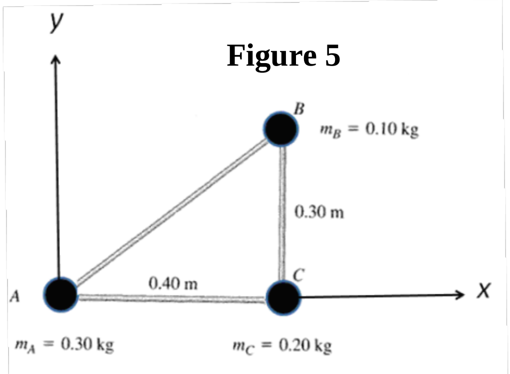
$$= 38.5 - 4.9 = 33.6\ \text{rad} = 5.3476\ \text{rev}$$

$$\Delta\theta_{above-x-axis} = 5.3476 - 5 = 0.3476\ \text{rev} = 2.18\ \text{rad}$$

Q7.

Three point masses m_A, m_B and m_C are located in the x-y plane at points A, B and C as shown in **Figure 5**. They are connected together by massless rods. Find the kinetic energy of the system if it rotates with an angular speed of 4.0 rad/s about the z-axis passing through point A.

A) 0.46 J
B) 0.69 J
C) 0.21 J
D) 1.2 J
E) 2.5 J



Ans:

$$K_{rot} = \frac{1}{2}I\omega^2$$

$$I = 0.1 \times (0.5)^2 + 0.2 \times (0.4)^2 = 0.057\ kg.m^2$$

$$K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.057 \times (4)^2$$

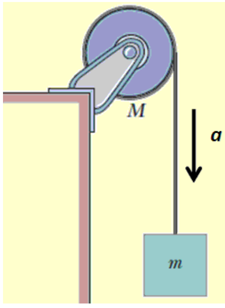
$$= 0.456 = 0.46\ J$$

Q8.

Figure 6 shows a uniform solid disk, with mass M and radius R = 0.280 m, mounted on a fixed horizontal axle. A block with mass m = 4.20 kg hangs from a massless cord that is wrapped around the rim of the disk. The system is released from rest and the block moves vertically downward with a constant acceleration of 1.5 m/s^2. Calculate the mass M of the disk. Assume the cord does not slip, and there is no friction at the axle.

A) 46.5 kg
B) 32.6 kg
C) 12.7 kg
D) 50.2 kg
E) 21.6 kg

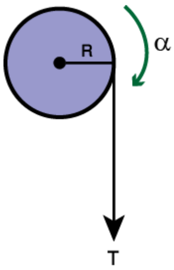
Figure 6



Ans:

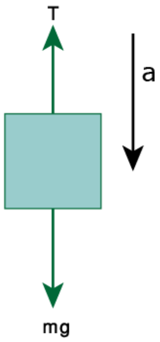
For the Pulley

$$T\cancel{R} = I\alpha = \frac{M\cancel{R^2}^{R}}{2} \times \frac{a}{\cancel{R}} = \frac{M\cancel{R}a}{2}$$



$$T = \frac{Ma}{2}$$

Then for the block



$$T - mg = -ma$$

$$\frac{Ma}{2} - mg = -ma$$

$$M = \frac{2m}{a}(g - a)$$

$$= \frac{2 \times 4.2}{1.5}(9.8 - 1.5) = 46.48 \text{ kg}$$

$$M = 46.5 \text{ kg}$$

Q9.

Consider a solid uniform disk of mass 45.0 kg and radius 1.30 m. It is rotating at 31.5 rad/s about a vertical frictionless axis passing through its center. What average power is required to bring the disk to a stop in 20.0 s.

A) 943 W
B) 458 W
C) 287 W
D) 854 W
E) 200 W

Ans:

$$P_{avg} = \frac{W}{t} = \frac{\Delta K_{rot}}{t} = \frac{\frac{1}{2} I\omega^2}{t} = \frac{\frac{1}{2} \times \frac{MR^2}{2} \times \omega^2}{t}$$

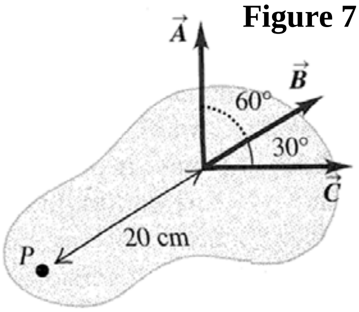
$$P_{avg} = \frac{\frac{1}{2} \times 45 \times \frac{(1.3)^2}{2} \times (31.5)^2}{20} = 943.3\ \mathrm{W}$$

Q10.

As shown in **Figure 7**, three forces $\vec{A}$, $\vec{B}$ and $\vec{C}$ with equal magnitude of 50.0 N act at the same point on the object. All forces are in the plane of the paper. What is the net torque about pivot point P?

A) 3.66 N·m out of the page
B) 7.35 N·m out of the page
C) 3.66 N·m into the page
D) 7.35 N·m into the page
E) 5.55 N·m out of the page

Figure 7



Ans:

$$\vec{\tau}_{net} = 0.2 \times (\vec{A}\sin 60 - \vec{C}\sin 30)$$

$$|A| = |C|$$

$$\vec{\tau}_{net} = 0.2 \times 50(\sin 60 - \sin 30)$$

$$\vec{\tau}_{net} = +3.66\ \mathrm{N.m}$$

Q11.

Consider a solid uniform sphere of radius R and mass M rolling along a horizontal floor without slipping. Find the ratio of its translational kinetic energy to its rotational kinetic energy.

A) 2.5
B) 0.5
C) 2.0
D) 1.0
E) 3.0

Ans:

$$K_{trans} = \frac{1}{2} M v_{com}^2$$

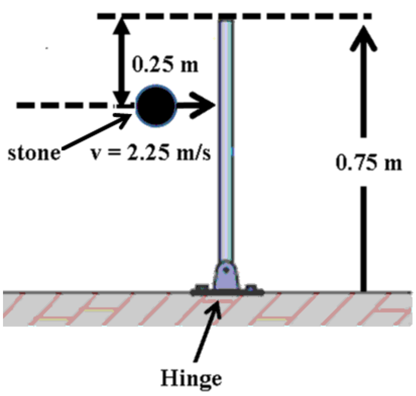
$$K_{rot} = \frac{1}{2} I\omega^2 = \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{5} M\cancel{R}^2 \times \left(\frac{v_{com}}{\cancel{R}}\right)^2 = \frac{1}{5} M v_{com}^2$$

$$\text{ratio } R = \frac{K_{trans}}{K_{rot}} = \frac{\frac{1}{2}\cancel{M}\,\cancel{v_{com}^2}}{\frac{1}{5}\cancel{M}\,\cancel{v_{com}^2}} = \frac{5}{2} = 2.5$$

Q12.

As shown in **Figure 8**, a 0.500 kg stone moving horizontally with a speed of 2.25 m/s collides with a 0.750 m long stationary vertical uniform rod. The rod has a mass of 1.50 kg and is fixed to the ground by a frictionless hinge. After the collision, the stone drops vertically down to the ground. What is the angular speed of the rod just after the collision?

Figure 8



A) 2.00 rad/s
B) 1.00 rad/s
C) 3.00 rad/s
D) 4.00 rad/s
E) 5.00 rad/s

Ans:

$$L_i = L_f$$

$$r(mv) = I\omega = \frac{ML^2}{3} \times \omega$$

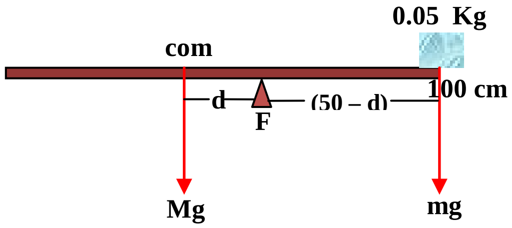
$$0.5 \times 0.5 \times 2.25 = \frac{1.5 \times (0.75)^2}{3} \times \omega$$

$$\omega = \frac{3 \times 0.5 \times 0.5 \times 2.25}{1.5 \times (0.75)^2} = 2.00 \text{ rad/s}$$

Q13.

A uniform 0.20 kg meter stick can be balanced horizontally on a knife edge if 0.050 kg point mass is placed at the 100-cm mark. Find the position of the knife edge measured from the zero-cm mark.

A) 60 cm
B) 70 cm
C) 50 cm
D) 40 cm
E) 80 cm



Ans:

Taking torque about knife edge F

$0.2 \times d \times \cancel{g} = 0.05 \times (50 - d) \times \cancel{g}$

$(0.2 + 0.05)d = 0.050 \times 50$

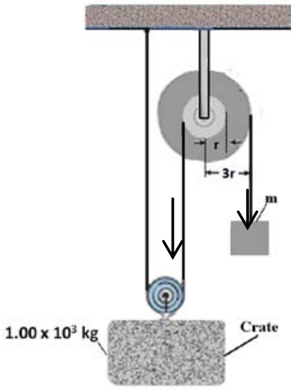
$d = 10\text{ cm}$

Position of knife edge from zero-cm mark = 50+10 = 60 cm

Q14.

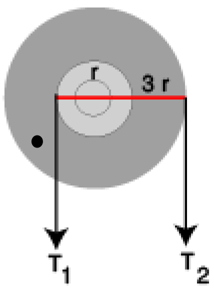
A system consisting of mass m, a crate and pulleys is shown in **Figure 9**. The pulleys are frictionless and massless. Find the value of mass m for which the system will be in static equilibrium.

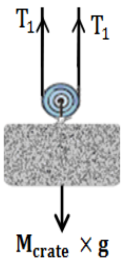
Figure 9



A) 167 kg
B) 344 kg
C) 259 kg
D) 433 kg
E) 101 kg

Ans:





$$\cancel{r}T_1 = 3\cancel{r}T_2$$

$$T_2 = \frac{T_1}{3}$$

$$m\cancel{g} = \frac{M_{crate} \times \cancel{g}}{2 \times 3}$$

$$m = \frac{M_{crate}}{6} = \frac{1000}{6} = 167\ \text{kg}$$

$$2T_1 = M_{crate} \times g$$

$$T_1 = \frac{M_{crate} \times g}{2}$$

$$T_2 = mg$$

Q15.

A 10.2 m long steel beam with a cross-sectional area of 0.120 m^2 is mounted between two concrete walls with no room for expansion. When the temperature rises, such a beam will expand in length by 1.20 mm if it is free to do so. What force must be exerted by the concrete walls to prevent the beam from expanding? Young's modulus for steel is 2.00×10^{11} N/m^2.

A) 2.83×10^{6} N
B) 4.01×10^{6} N
C) 3.67×10^{5} N
D) 1.45×10^{6} N
E) 2.09×10^{5} N

Ans:

$$F = E \times \frac{\Delta L}{L} \times A$$

$$= 2 \times 10^{11} \times \frac{1.20 \times 10^{-3}}{10.2} \times 0.120 = 2.82 \times 10^{6}\ \mathrm{N}$$