Q1.
A 10 kg box slides with a constant speed a distance of 5.0 m downward along a rough slope that makes an angle $\theta$ with the horizontal (see Figure 1). If the work done by the force of gravity is 360 J , the angle $\theta$ is:
A) $47^{\circ}$
B) $37^{\circ}$
C) $42^{\circ}$
D) $50^{\circ}$
E) $75^{\circ}$

Ans:
$\mathrm{m}=10 \mathrm{~kg}, \quad \mathrm{~d}=5.0 \mathrm{~m}$
$\mathrm{W}_{\mathrm{mg}}=360 \mathrm{~J}$
$\therefore \mathrm{W}_{\mathrm{mg}}=\mathrm{mgh}=\mathrm{mgdsin} \theta$
$\sin \theta=\frac{W_{\mathrm{mg}}}{\mathrm{mgd}}=\frac{360}{10 \times 9.8 \times 5}$
$\sin \theta=0.735$
$\theta=47.28^{\circ} \approx 47^{\circ}$

Q2.
A mechanic pushes a $2.00 \times 10^{3} \mathrm{~kg}$ car horizontally a distance of 4.00 m from rest to a speed of $4.00 \mathrm{~m} / \mathrm{s}$ with a constant force. Find the magnitude of the net force acting on the car.
A) $4.00 \times 10^{3} \mathrm{~N}$
B) $3.00 \times 10^{3} \mathrm{~N}$
C) $4.00 \times 10^{4} \mathrm{~N}$
D) $2.00 \times 10^{2} \mathrm{~N}$
E) $9.00 \times 10^{2} \mathrm{~N}$

Ans:

$$
\begin{aligned}
& \Delta \mathrm{K}=\mathrm{W} \Rightarrow \mathrm{Fd}=\frac{1}{2} \mathrm{mv}^{2}-0 \\
& \therefore \mathrm{~F}=\frac{\mathrm{mv}^{2}}{2 \mathrm{~d}}=\frac{2 \times 10^{3} \times 1 / 6^{4}}{2 \times \not{ }^{4}} \\
& =4.00 \times 10^{3} \mathrm{~N}
\end{aligned}
$$

Q3.
A person pulls horizontally a 20 kg box at a constant speed of $3.0 \mathrm{~m} / \mathrm{s}$ for 10 s . The coefficients of kinetic and static friction between the box and the horizontal floor are 0.30 and 0.60 , respectively. What is the work done by the person in pulling the box?
A) 1.8 kJ
B) 3.6 kJ
C) -1.8 kJ
D) -3.6 kJ
E) 1.5 kJ

Ans:
$\mathrm{W}_{\text {net }}=\mathrm{W}_{\mathrm{F}}+\mathrm{W}_{\mathrm{f}}=\Delta \mathrm{K}=0 ; \quad \mathrm{d}=30 \mathrm{~m}$
$\therefore \mathrm{W}_{\mathrm{F}}=-\mathrm{W}_{\mathrm{f}}=+\mu_{\mathrm{k}} \mathrm{mgd}=0.3 \times 20 \times 9.8 \times 30$
$=1764.0 \mathrm{~J}=1.8 \mathrm{~kJ}$

## Q4.

A machine applies a constant force $\overrightarrow{\mathrm{F}}=(3.00 \mathrm{~N}) \hat{i}+(2.00 \mathrm{~N}) \hat{j}+(5.00 \mathrm{~N}) \hat{k}$ on a 4.00 kg box. The box is carried from an initial position o $\overrightarrow{\mathrm{d}}_{\mathrm{i}}=(1.00 \mathrm{~m}) \hat{i}+(2.00 \mathrm{~m}) \hat{j}-(2.00 \mathrm{~m}) \hat{k}$ to a final position of $\overrightarrow{\mathrm{d}}_{\mathrm{f}}=(6.00 \mathrm{~m}) \hat{i}-(3.00 \mathrm{~m}) \hat{j}-(2.00 \mathrm{~m}) \hat{k}$ in 12.0 s . Find the average power of the machine's force on the box in this time interval of 12.0 s .
A) 0.417 W
B) 0.231 W
C) 3.45 W
D) 0.864 W
E) 2.34 W

Ans:

$$
\begin{aligned}
& \mathrm{P}_{\mathrm{avg}}=\frac{\mathrm{W}}{\mathrm{t}}=\frac{\overrightarrow{\mathrm{F}} \cdot \overrightarrow{\mathrm{~d}}}{\mathrm{t}}=\frac{15-10}{12}=\frac{5}{12}=0.417 \mathrm{~W} \\
& \overrightarrow{\mathrm{~F}}=(3.0) \hat{\imath}+(2.0) \hat{\jmath}+(5.0) \hat{k} \\
& \overrightarrow{\mathrm{~d}}=(5.0) \hat{\imath}-(5.0) \hat{\jmath}
\end{aligned}
$$

Q5.
A block of mass $m=12.0 \mathrm{~kg}$ is released from rest on a frictionless incline of angle $\theta=$ $30.0^{\circ}$ (see Figure 2). Below the block is a spring that can be compressed 2.50 cm by a force of 250 N . The block stops momentarily when it compresses the spring by 4.00 cm . How far does the block move down the incline from its rest position to this stopping point?
A) 13.6 cm
B) 17.0 cm
C) 8.00 cm
D) 4.50 cm
E) 6.50 cm

Ans:

$$
\mathrm{K}=\frac{250}{2.50 \times 10^{-2}}=10^{4} \mathrm{~N} / \mathrm{m}
$$

$$
\Delta \stackrel{0}{0}+\Delta U_{g}+\Delta U_{s}=0
$$

$$
-\mathrm{mgd} \sin \theta+\frac{1}{2} \mathrm{Kx}^{2}=0
$$

$$
d=\frac{K x^{2}}{2 \mathrm{mg} \sin \theta}=\frac{10^{4} \times(4)^{2} \times 1,0^{-4}}{2 \times 12 \times 9.8 \times 0.5}=0.136 \mathrm{~m}=13.6 \mathrm{~cm}
$$

Q6.
A worker does 200 J of work in moving a 20.0 kg box a distance $D$ on a rough horizontal floor. The box starts from rest and its speed after moving the distance $D$ is $4.00 \mathrm{~m} / \mathrm{s}$. Find the work done by the force of friction.
A) -40.0 J
B) 40.0 J
C) -200 J
D) 200 J
E) 160 J

Ans:

$$
\mathrm{W}_{\mathrm{net}}=\Delta \mathrm{K}=\mathrm{W}_{\mathrm{App}}+\mathrm{W}_{\mathrm{f}}
$$

$\frac{1}{2} \mathrm{mv}^{2}=200+\mathrm{W}_{\mathrm{f}}$
$\mathrm{W}_{\mathrm{f}}=\frac{1}{2} \mathrm{mv}^{2}-200=\frac{1}{2} \times 20^{10} \times 16-200=160-200=-40 \mathrm{~J}$

Q7.
In Figure 3, a moving block can take three frictionless paths, differing only in elevation (height), to reach the dashed finish line. Rank the paths according to the speed of the block at the finish line, greatest first.

Figure 3
А) $3,2,1$
B) $1,2,3$
C) $2,3,1$
D) $1,3,2$
E) 2,1,3


Ans:
A
Q8.
Figure 4 shows three identical uniform bars, each of mass $M$ and length 2R, welded to a uniform ring of mass 2 M and radius R . The bars and the ring are in an $x y$-plane whose origin O is at the center of the ring. Find the $x$ and $y$ coordinates of the system's center of mass.
A) $(0.2 \mathrm{R}, 0.4 \mathrm{R})$
B) $(0.5 \mathrm{R}, 0.3 \mathrm{R})$
C) $(0.5 \mathrm{R}, 0.5 \mathrm{R})$
D) $(0.2 \mathrm{R}, 0.5 \mathrm{R})$
E) ( $0.5 \mathrm{R}, 0.2 \mathrm{R}$ )

Ans:
$\mathrm{x}_{\mathrm{cm}}=\frac{M / R}{5 M}=0.2 \mathrm{R}$
$y_{c m}=\frac{M \times 2 R}{5 R^{\prime}}=0.4 \mathrm{R}$


Q9.
Consider a one dimensional collision between two identical balls. One is originally at rest and the other has a velocity of $(4 \mathrm{~m} / \mathrm{s}) \hat{i}$. If $3 / 8$ of the initial kinetic energy is lost during the collision, find the velocities of the balls after the collision.
A) $(1 \mathrm{~m} / \mathrm{s}) \hat{i}$ and $(3 \mathrm{~m} / \mathrm{s}) \hat{i}$
B) $(1 \mathrm{~m} / \mathrm{s}) \hat{i}$ and $(5 \mathrm{~m} / \mathrm{s}) \hat{i}$
C) $(2 \mathrm{~m} / \mathrm{s}) \hat{i}$ and $(6 \mathrm{~m} / \mathrm{s}) \hat{i}$
D) $(2 \mathrm{~m} / \mathrm{s}) \hat{i}$ and $(2 \mathrm{~m} / \mathrm{s}) \hat{i}$
E) 0 and $(4 \mathrm{~m} / \mathrm{s}) \hat{i}$

Ans:
$\mathrm{m}_{1} \stackrel{\rightharpoonup}{\mathrm{v}}_{1 \mathrm{i}}+\mathrm{m}_{2} \overrightarrow{\mathrm{t}}_{2 \mathrm{i}}=\mathrm{m}_{1} \mathrm{t}_{1 \mathrm{f}}+\mathrm{m} / \overrightarrow{\mathrm{t}}_{2 \mathrm{f}} ; \mathrm{m}_{1}=\mathrm{m}_{2}$
$4 \hat{\imath}=\vec{v}_{1 \mathrm{f}}+\overrightarrow{\mathrm{v}}_{2 \mathrm{f}} \Rightarrow 4=\mathrm{v}_{1 \mathrm{f}}+\mathrm{v}_{2 \mathrm{f}}$
$\frac{5}{\phi}\left(\frac{1}{2} n k \times 1 \frac{2}{\phi}\right)=\frac{1}{2} n / v_{1 f}^{2}+\frac{1}{k} h v_{2 f}{ }^{2}$
$10=v_{1 f}{ }^{2}+v_{2 f}^{2}=v_{2 f}^{2}+\left(4-v_{1 f}\right)^{2}$
$10=\mathrm{v}_{1 \mathrm{f}}{ }^{2}+16+\mathrm{v}_{1 \mathrm{f}}{ }^{2}-8 \mathrm{v}_{1 \mathrm{f}}$
$2 \mathrm{v}_{1 \mathrm{f}}{ }^{2}-8 \mathrm{v}_{1 \mathrm{f}}+6=0$
$\mathrm{v}_{1 \mathrm{f}}=\frac{8 \pm \sqrt{64-48}}{4}=\frac{8 \pm 4}{4}=\frac{12}{4}=3$ or 1

Q10.
A ball of mass m, suspended by a light string of length 100 cm is released from a position where the string makes an angle $\theta=60.0^{\circ}$ with the vertical (see Figure 5). The ball collides with a second identical ball kept at rest on a smooth wedge at the bottom of its swing and the two balls stick together after collision. Determine the maximum vertical height H to which the balls rise after the collision.

Figure 5
A) 12.5 cm
B) 25.0 cm
C) 50.0 cm
D) 33.5 cm
E) 42.5 m

$\mathrm{v}_{1}=\sqrt{2 \mathrm{gL}(1-0.5)}=3.13 \mathrm{~m} / \mathrm{s}$
$m n_{1} v_{1 i}+m v_{2 i}=\left(m_{1}+m_{2}^{2}\right) v_{f}$
$3.13=2 \mathrm{v}_{\mathrm{f}} \Rightarrow \mathrm{v}_{\mathrm{f}}=1.565$
$\therefore 2 \mathrm{n} / \mathrm{gH}=\frac{1}{2}(2 \not 2 \mathrm{n})(1.565)^{2}$
$H={\frac{(1.565)^{2}}{2 g}}_{2}=0.125 \mathrm{~m}$

## Q11.

Ball A, moving with velocity $(8 \mathrm{~m} / \mathrm{s}) \hat{i}$, collides with stationary Ball B. Masses of balls A and $\mathbf{B}$ are the same. After the collision, if $\mathbf{B}$ moves with velocity $(6 \mathrm{~m} / \mathrm{s}) \hat{j}$, what is the velocity of $\mathbf{A}$ ?
A) $(8 \mathrm{~m} / \mathrm{s}) \hat{i}-(6 \mathrm{~m} / \mathrm{s}) \hat{j}$
B) $(8 \mathrm{~m} / \mathrm{s}) \hat{i}+(6 \mathrm{~m} / \mathrm{s}) \hat{j}$
C) $-(8 \mathrm{~m} / \mathrm{s}) \hat{i}-(6 \mathrm{~m} / \mathrm{s}) \hat{j}$
D) $-(8 \mathrm{~m} / \mathrm{s}) \hat{i}+(6 \mathrm{~m} / \mathrm{s}) \hat{j}$
E) $-(6 \mathrm{~m} / \mathrm{s}) \hat{j}$

Ans:

$$
\begin{aligned}
& m / \vec{v}_{A i}+m / /_{\mathrm{B}} \vec{v}_{B i}=m /{ }_{\mathrm{A}} \overrightarrow{\mathrm{v}}_{\mathrm{Af}}+\mathrm{m}_{\mathrm{B}} \vec{v}_{\mathrm{Bf}} \\
& (8 \mathrm{~m} / \mathrm{s}) \hat{\imath}+0=\vec{v}_{\mathrm{Af}}+(6 \mathrm{~m} / \mathrm{s}) \hat{\jmath} \\
& \overrightarrow{\mathrm{v}}_{\mathrm{Af}}=(8 \mathrm{~m} / \mathrm{s}) \hat{\imath}-(6 \mathrm{~m} / \mathrm{s}) \hat{\jmath}
\end{aligned}
$$

## Q12.

A 2.0 kg toy car moves along an x-axis. Figure 6 shows the net force along the x-axis acting on the car as a function of time. The car begins from rest at $t=0$. What is the velocity of the car at $t=9 \mathrm{~s}$ ?

Figure 6

A) $(15 \mathrm{~m} / \mathrm{s}) \hat{i}$
B) $(25 \mathrm{~m} / \mathrm{s}) \hat{i}$
C) $-(15 \mathrm{~m} / \mathrm{s}) \hat{i}$
D) $-(25 \mathrm{~m} / \mathrm{s}) \hat{i}$
E) $(10 \mathrm{~m} / \mathrm{s}) \hat{i}$

Ans:

$$
\begin{aligned}
\Delta \mathrm{p}=\mathrm{mv}_{\mathrm{f}}= & \frac{1}{5}\left(\frac{\mathrm{q}}{\mathrm{q}}\right)(10)+2 \times 10+\frac{1}{5} \times \frac{2}{4} \times 10 \\
& +\left(-\frac{1}{2} \times 1 \times 5\right)-5 \times 1-\frac{1}{2} \times 1 \times 5 \\
= & 10+20+10-5 .-\frac{5}{2} \\
\operatorname{mv}_{\mathrm{f}}= & 30 \Rightarrow \mathrm{v}_{\mathrm{f}}=(15 \mathrm{~m} / \mathrm{s}) \hat{\imath}
\end{aligned}
$$

Q13.
A disk, initially at rest, is rotated with constant angular acceleration $\alpha=5.00 \mathrm{rad} / \mathrm{s}^{2}$ for 8.00 s . The disk is then brought to rest with uniform negative acceleration in 10.0 revolutions. Determine the magnitude of the negative acceleration required.
A) $12.7 \mathrm{rad} / \mathrm{s}^{2}$
B) $15.0 \mathrm{rad} / \mathrm{s}^{2}$
C) $10.4 \mathrm{rad} / \mathrm{s}^{2}$
D) $1.25 \mathrm{rad} / \mathrm{s}^{2}$
E) $16.0 \mathrm{rad} / \mathrm{s}^{2}$

Ans:

$$
\omega_{\mathrm{i}}=0+5 \times 8=40 \mathrm{rad} / \mathrm{s}
$$

${ }^{0}{ }^{2}-\omega_{\mathrm{i}}{ }^{2}=2 \alpha \Delta \theta$
$-40 \times 40=2 \alpha \times 10 \times 2 \pi$
$\alpha=-\frac{40 \times 40}{40 \pi}=-12.72 \mathrm{rad} / \mathrm{s}^{2}$

## Q14.

Figure 7 shows a uniform metal plate that had been square before $25 \%$ of it was removed. Three lettered points are indicated. Rank them according to the rotational inertia of the plate about a perpendicular axis through them, greatest first.
A) c, a, b
B) c, a \& b tie
C) a, b \& c tie
D) $\mathrm{a}, \mathrm{c}, \mathrm{b}$
E) b, a, c

Ans:
A


Q15.
A $1.0 \times 10^{3} \mathrm{~N}$ man is standing $6.0 \times 10^{3} \mathrm{~km}$ to the East from the center of earth. Find the torque about the center of the earth exerted by the force of gravity on the man. (Assume the Earth is a sphere).
A) 0
B) $6.0 \times 10^{9} \mathrm{~N} . \mathrm{m}$ North
C) $6.0 \times 10^{9}$ N.m South
D) $6.0 \times 10^{9} \mathrm{~N} . \mathrm{m}$ East
E) $6.0 \times 10^{9} \mathrm{~N} . \mathrm{m}$ West

Ans:

## A

Q16.
Figure 8 shows a uniform disk that can rotate around its center. The disk has a radius of 4.00 cm and a mass of 40.0 grams and is initially at rest. Starting at $t=0$, two forces are to be applied tangentially to the rim as indicated, so that at time $t=2.50 \mathrm{~s}$ the disk has angular velocity of $250 \mathrm{rad} / \mathrm{s}$ clockwise. Force $\mathrm{F}_{1}$ has a magnitude of 0.100 N . What is the magnitude of $\mathrm{F}_{2}$ ?

Figure 8
A) $2.00 \times 10^{-2} \mathrm{~N}$
B) $6.00 \times 10^{-2} \mathrm{~N}$
C) 1.50 N
D) $1.06 \times 10^{-1} \mathrm{~N}$
E) $4.20 \times 10^{-1} \mathrm{~N}$


Ans:

$$
\begin{aligned}
& \mathrm{F}_{1} \mathrm{R}-\mathrm{F}_{2} \mathrm{R}=\mathrm{I} \alpha \\
& \mathrm{I}=\frac{1}{2} \mathrm{MR}^{2}=\frac{1}{2} \times 40 \times 10^{-3} \times\left(4 \times 10^{-2}\right)^{2} \\
& =\frac{1}{2} \times 4 \times 10^{-3} \times 16 \times 10^{-4}=32 \times 10^{-7} \mathrm{~kg} \cdot \mathrm{~m}^{2} \\
& \alpha=\frac{\omega_{\mathrm{f}}-\omega_{\mathrm{i}}}{\mathrm{t}}=\frac{250}{2.5}=10^{2} \mathrm{rad} / \mathrm{s}^{2} \\
& \mathrm{~F}_{2}=\frac{\mathrm{F}_{1} \mathrm{R}-\mathrm{I} \alpha}{\mathrm{R}}=0.1-32 \times 10^{-7} \times 10^{2} / \mathrm{R} \\
& \mathrm{~F}_{2}=0.1-\frac{32 \times 10^{-5}}{4 \times 10^{-2}} \\
& \mathrm{~F}_{2}=0.1-8 \times 10^{-3}=0.1-0.08=0.02=2.00 \times 10^{-2} \mathrm{~N}
\end{aligned}
$$

## Q17.

A frictionless pulley has the shape of a uniform solid disk of mass 2.50 kg and radius 20.0 cm . A 1.50 kg stone is attached to a massless wire that is wrapped around the rim of the pulley (Figure 9), and the system is released from rest. How far must the stone fall so that the pulley has 4.50 J of kinetic energy?
A) 0.673 m
B) 0.543 m
C) 0.876 m
D) 0.954 m
E) 1.03 m

Ans:

$$
\begin{aligned}
& \frac{1}{2} \mathrm{I} \omega^{2}=4.50 \mathrm{~J} \\
& \frac{1}{2}\left(\frac{1}{2} \mathrm{MR}^{2} \omega^{2}\right)=\frac{1}{4} \mathrm{Mv}^{2}=4.50 \mathrm{~J} \\
& \mathrm{v}^{2}=\frac{4 \times 4.50}{2.5}=7.2 \mathrm{~m}^{2} / \mathrm{s}^{2} \\
& (\Delta \mathrm{~K})_{\text {mass }}+(\Delta \mathrm{K})_{\text {pulley }}+\left(\Delta \mathrm{U}_{\mathrm{g}}\right)_{\text {mass }}=0
\end{aligned}
$$

$$
\frac{1}{2} \mathrm{mv}^{2}+4.50 \mathrm{~J}-\mathrm{mgh}=0 \Rightarrow \mathrm{mgh}=+\frac{1}{2}\left(\mathrm{mv}^{2}\right)+4.50
$$

$$
=\frac{1}{2}(1.5)(7.2)+4.50=9.9
$$

$$
\mathrm{h}=\frac{9.9}{1.5 \times 9.8}=0.673 \mathrm{~m}
$$

Q18.
A student releases from rest a solid sphere (object P), a thin-walled hollow sphere (object Q), a solid cylinder (object R), and a thin-walled hollow cylinder (object S) from the same height at the top of an inclined plane (see Figure 10). Which one of these objects will reach the bottom of the incline first?

Figure 10
A) $P$
B) Q
C) $R$
D) S
E) They all take the same time

Ans:
A


Q19.
Figure 11 shows three particles of the same mass and the same constant speed moving as indicated by the velocity vectors. Points a, b, c, and d form a square. Rank the magnitudes of the net angular momenta of the three-particle system about these points, $L_{a}, L_{b}, L_{c}$ and $L_{d}$, greatest first.

Figure 11
A) $\mathrm{L}_{\mathrm{a}}, \mathrm{L}_{\mathrm{b}}$ and $\mathrm{L}_{\mathrm{c}}$ tie, $\mathrm{L}_{\mathrm{d}}$
B) $\mathrm{L}_{\mathrm{b}}, \mathrm{L}_{\mathrm{a}}$ and $\mathrm{L}_{\mathrm{c}}$ tie, $\mathrm{L}_{\mathrm{d}}$
C) All tie
D) $\mathrm{L}_{\mathrm{b}}, \mathrm{L}_{\mathrm{d}}, \mathrm{L}_{\mathrm{a}}$ and $\mathrm{L}_{\mathrm{c}}$ tie
E) $L_{d}, L_{a}, L_{c}$ and $L_{b}$ tie

Ans:
A


## Q20.

Figure 12 shows two disks A \& B rotating in the same direction. Disk A has a mass of 2.00 kg , a radius of 0.200 m , and an initial angular speed of $50.0 \mathrm{rad} / \mathrm{s}$. Disk B has a mass of 4.00 kg , a radius of 0.100 m , and an initial angular speed of $200 \mathrm{rad} / \mathrm{s}$. The disks are pushed toward each other with equal and opposite forces acting along the common axis. Find the common angular speed after the disks are pushed together into contact.

Figure 12
A) $100 \mathrm{rad} / \mathrm{s}$
B) $120 \mathrm{rad} / \mathrm{s}$
C) $87.0 \mathrm{rad} / \mathrm{s}$
D) $95.0 \mathrm{rad} / \mathrm{s}$
E) $65.0 \mathrm{rad} / \mathrm{s}$

Ans:

$$
\begin{aligned}
& \mathrm{I}_{1} \omega_{1}+\mathrm{I}_{2} \omega_{2}=\left(\mathrm{I}_{1}+\mathrm{I}_{2}\right) \omega \\
& \omega=\frac{\mathrm{I}_{1} \omega_{1}+\mathrm{I}_{2} \omega_{2}}{\left(\mathrm{I}_{1}+\mathrm{I}_{2}\right)} \\
& \mathrm{I}_{1}=\frac{1}{2}(2)(0.2)^{2}=0.04 \mathrm{~kg} \cdot \mathrm{~m}^{2} \\
& \mathrm{I}_{2}=\frac{1}{2}(4)(0.1)^{2}=0.02 \mathrm{~kg} \cdot \mathrm{~m}^{2} \\
& \begin{aligned}
& \omega_{1}=50 \mathrm{rad} / \mathrm{s}, \omega_{2}=200 \mathrm{rad} / \mathrm{s} \\
& \begin{aligned}
& =\frac{0.04 \times 50+0.02 \times 200}{0.06} \\
& =\frac{2.0+4.0}{0.06} \\
& =100 \mathrm{rad} / \mathrm{s}
\end{aligned}
\end{aligned} .
\end{aligned}
$$

