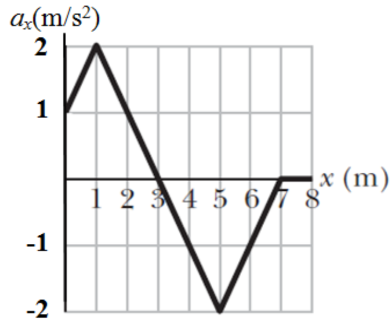
Q1.

Figure 1 shows a graph of the acceleration versus the displacement of a particle moving in one dimension. The particle is at rest at $x = 0$. What is the coordinate x at which the particle has the maximum kinetic energy?

Figure 1



A) 3
B) 1
C) 5
D) 7
E) 8

Ans:

$\Delta K = W =$ area under the curve

Q2.

What is the work done by the force $\vec{F} = (2.0\hat{i} + 4.0\hat{j} + 9.0\hat{k})$ N that acts on a 3.0 kg object and moves from an initial position $\vec{r}_1 = (-4.1\hat{i} + 3.3\hat{j} + 5.2\hat{k})$ m to a final position $\vec{r}_2 = (2.7\hat{i} - 2.9\hat{j} + 5.5\hat{k})$ m.

A) -8.5 J
B) -12 J
C) 5.2 J
D) 12 J
E) 4.7 J

Ans:

$$\vec{d} = \vec{r}_2 - \vec{r}_1$$

$$W = \vec{F} \cdot \vec{d}$$

Q3.

A 25 kg block, which is initially at rest, is pulled across a horizontal rough surface (coefficient of kinetic friction $\mu_k = 0.3$) by a force of 80 N directed 30° above the horizontal. What is the final speed of the block after it has moved a distance d = 10 m?

A) 2.5 m/s
B) 1.8 m/s
C) 3.3 m/s
D) 4.0 m/s
E) 3.8 m/s

Ans:

$$\Delta k = W = W_F + W_f$$

$$W_F = (80\cos 30°)d = 692.8 \text{ J}$$

$$W_F = -\mu F_N d = \mu(mg - 80 sin30)d = -615 \text{ J}$$

$$K_f = \frac{1}{2} m v_f^2 = W_F + W_f = 77 \text{ J} \Rightarrow v_f = 2.5\ m/s$$

Q4.

A worker uses a motor to raise a 2700-kg block a vertical distance of 3.2 m. If the task is to be achieved in 5.0 minutes, what is the minimum (average) power requirement for the motor?

A) 280 W
B) 320 W
C) 440 W
D) 510 W
E) 560 W

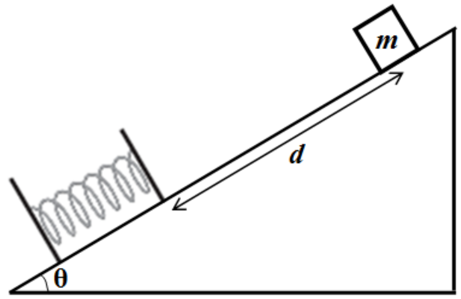
Ans:

$$P = \frac{W}{t} = \frac{mgh}{t} = 282 \text{ W}$$

Q5.

A block of mass $m = 3.5$ kg slides from rest down a frictionless incline of angle $\theta = 30°$. See **Figure 2**. After sliding a distance d along the incline, it compresses a relaxed spring of spring constant 430 N/m. The block monetarily stops after compressing the spring by 20 cm. What is the distance d ?

Figure 2



A) 30 cm
B) 40 cm
C) 50 cm
D) 55 cm
E) 60 cm

Ans:

Conservation of energy:

$$mg(d + X)sin\theta = \frac{1}{2}hx^2$$

$$d + X = 0.50\ m$$

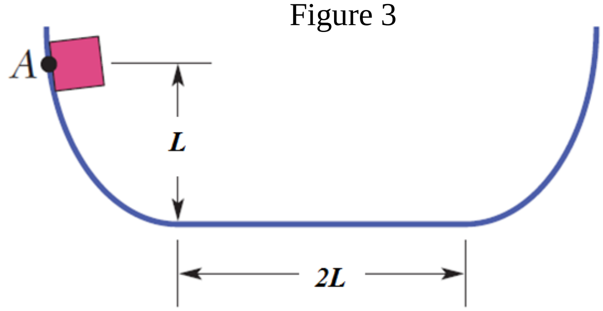
$$d = 0.30\ m$$

Q6.

A box can slide along a track with elevated curved ends and a flat central part, as shown in **Figure 3**. The flat part has length $2L$. The curved portions of the track are frictionless, but for the flat part the coefficient of kinetic friction is $\mu_k = 0.25$. The box is released from rest at point A on the left curved portion, which is at a height L. Find the maximum height that the box will reach at the right curved portion.

A) $L/2$
B) L
C) $3L/2$
D) $2L/3$
E) $L/3$

Figure 3



Ans:

Conservation of energy:

$$mgL' = mgL - \mu mg(2L)$$

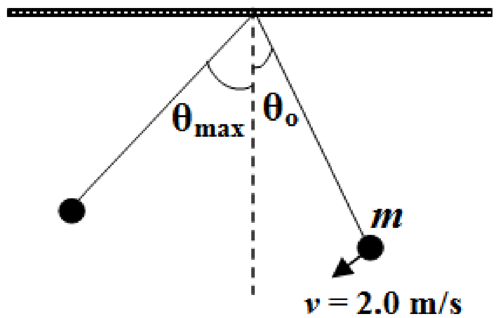
$$L' = L - \mu(2L) = 0.5\ L$$

Q7.

A pendulum consists of a small mass m = 0.15 kg attached to a massless rod with length 2.0 m. At θ_o = 15°, the mass has a speed of 2.0 m/s (see **Figure 4**). Find the maximum angle θ_{max} that the pendulum will make with the vertical. (Ignore air resistance)

A) 30°
B) 37°
C) 27°
D) 45°
E) 60°

Figure 4



Ans:

U = 0 at initial position
H_i = height at initial position
H_f = height at final position

H= H_i - H_f

$$\frac{1}{2}mv_i^2 = mgh \Rightarrow h = 0.2\,m$$

$$H_i = Lcos\theta_0, H_f = Lcos\theta_{max}$$

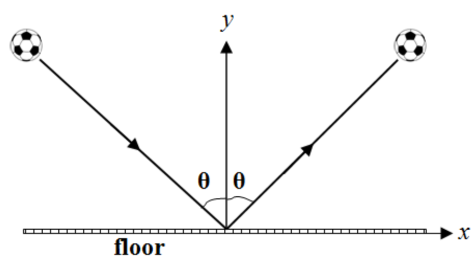
$$\frac{h}{L} = \frac{H_i - H_f}{L} = cos\theta_0 - cos\theta_{max} \Longrightarrow \theta_{max} = 30°$$

Q8.

A ball of mass 0.15 kg and with initial speed 2.0 m/s, collides elastically with the floor at θ = 30° with the vertical and bounces back with the same speed and the same angle (see **Figure 5**). What is the impulse from the ball on the floor?

A) 0.52 N.s along the negative y-axis
B) 0.52 N.s along the positive y-axis
C) 0.30 N.s along the positive x-axis
D) 0.30 N.s along the negative x-axis
E) 0

Figure 5



Ans:

Impulse on the bell

$$\vec{J} = m\left(\vec{v}_f - \vec{v}_i\right)$$

$$= m\left(2\cos\theta\hat{j} - (-2\cos\theta\hat{j})\right) = 4m\cos\theta\hat{j} = 0.52\,\hat{j}$$

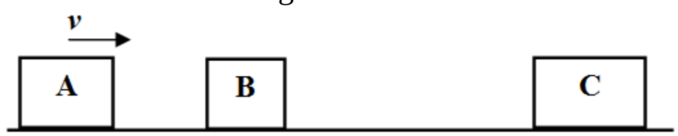
$$Impulse\ on\ the\ floor\ is\ -0.52\,\hat{j}$$

Q9.

Figure 6, shows three blocks A, B, and C, of masses 3*M*, *M* and 2*M* respectively, on a frictionless surface. Blocks B and C are initially at rest. Block A is initially moving towards block B at a speed *v* and undergoes an elastic collision with block B. Block B moves to the right and undergoes a completely inelastic collision with block C. What is the speed of block C immediately after the collision?

A) *v*/2
B) *v*/3
C) 3*v*/4
D) *v*/4
E) 2*v*/3

Figure 6



Ans:

Elastic collision between A and B gives:

$$v_B = \frac{2m_A}{m_A + m_B}\, v = \frac{6m}{4m} v = \frac{3}{2}\, v$$

Inelastic collision:

$$m_B v_B = (m_B + m_c)V \Rightarrow V = \frac{m_B}{m_B + m_c}\left(\frac{3}{2} v\right) = v/2$$

Q10.

Object A of mass *m* is moving along the *x*-axis with an initial speed of 5.0 m/s. It then collides with another object B of mass 4*m*, which is initially at rest. After the collision, object A moves with a speed of 2.5 m/s in a direction 60° to its original direction of motion. If the collision is inelastic, determine the direction of travel of object B with respect to the *x*-axis.

A) 30°
B) 37°
C) 60°
D) 45°

Ans:

Conservation of momentum along x-axis:

$$m_1 v_{1i} = m_1 v_{1f} cos\theta_1 + m_2 v_{2f} cos\theta_2$$

$$5 = 2.5cos60° + 4v_{2f} cos\theta_2 \Rightarrow v_{2f} cos\theta_2 = 0.9375$$

Conservation of momentum along y-axis:

$$m_1 v_{1f} sin60° = m_2 v_{2f} sin\theta$$

$$2.5sin60° = 4v_{2f} sin\theta_2 \Longrightarrow v_{2f} = 0.54\ sin\theta_2$$

$$\frac{0.9375}{cos\theta_2} = 0.54 sin\theta_2$$

$$tan\theta_2 = \frac{0.54}{0.9375} = 0.576$$

$$\Rightarrow \theta = 29.9°$$

Q11.

A stationary block lying on a frictionless floor explodes into three pieces that slide across the floor. Which one of the following quantities **will NOT be zero** after the explosion?

A) The total kinetic energy
B) The velocity of the center of mass
C) The acceleration of the center of mass
D) The total linear momentum
E) The displacement of the center of mass

Ans:

A

Q12.

A car and a truck, initially at rest at a traffic light, start accelerating in the same direction when the traffic light turns green. The acceleration of the car is 15.0 m/s^2 and the acceleration of the truck is 5.0 m/s^2. If the mass of the truck is three times the mass of the car, then what is the magnitude of the velocity of their center of mass after 4.0 s?

A) 30 m/s
B) 20 m/s
C) 40 m/s
D) 25 m/s
E) 35 m/s

Ans:

$$V_{car} = V_0 + at = 60\ m/s$$

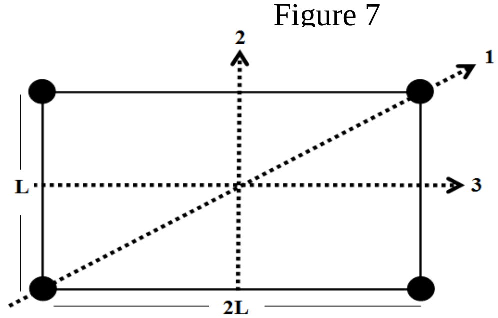
$$V_{truck} = 20\ m/s$$

$$V_{cm} = \frac{40 + 3 \times 20}{4} = 30\,m/s$$

Q13.

Four equal masses are arranged at the corners of a rectangle of width *L* and length 2*L*. The masses are connected by rigid, massless rods. The system can rotate about any one of the axes 1, 2, or 3 shown in **Figure 7**. Rank the axes according to the value of the rotational inertia about them, **greatest to least**.

Figure 7



A) 2, 1, 3
B) 2, 3, 1
C) 1, 2, 3
D) 1, 3, 2
E) 3, 2, 1

Ans:

(1) $2md^2 = 2mL^2 sin^2\theta = 1.6\,mL^2$

(2) $4mL^2$

(3) $4m\left(\frac{L}{2}\right)^2 = mL^2$

Q14.

A rotating ring has a radius R = 22 m. The angular position of a reference line on the ring is given by: $\theta(t) = 1.5t^3 - 4.0t^2$. Find the ratio of the magnitude of the tangential acceleration to the magnitude of the radial acceleration at t = 2.0 s for a point on the rim of the ring.

A) 2.5
B) 3.0
C) 4.0
D) 5.5
E) 4.4

Ans:

$$\omega(t) = 4.5\,t^2 - 8t \implies \omega(2) = 2\ rad/s$$

$$\alpha(t) = 9t - 8 \implies \alpha(2) = 10\,rad/s^2$$

$$\frac{a_t}{a_r} = \frac{r\alpha}{r\omega^2} = \frac{\alpha}{r\omega^2} = \frac{10}{4} = 2.5$$

Q15.

A wheel is initially rotating at 10 rad/s and has a constant angular acceleration. After 9.0 s, it has rotated through 24 revolutions. What is the magnitude of its angular acceleration?

A) 1.5 rad/s^2
B) 3.5 rad/s^2
C) 4.4 rad/s^2
D) 5.3 rad/s^2
E) 5.9 rad/s^2

Ans:

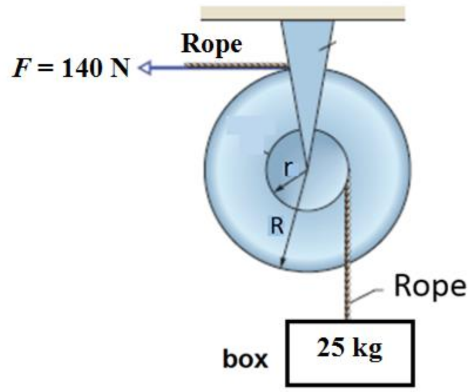
$$\theta = \omega \cdot t + \frac{1}{2}\alpha\, t^2$$

$$\alpha = \frac{2(\theta - \omega_0 t)}{t^2} = 1.5\ rad/s^2$$

Q16.

A yo-yo shaped device mounted on a horizontal frictionless axis is used to lift a 25 kg box as shown in **Figure 8**. The box is suspended by a rope wrapped around the axle whose radius *r* is 0.30 m. The outer radius *R* of the device is 0.6 m. When a constant horizontal force *F* of magnitude 140 N is applied to the rope at the outer radius of the device, the box has an upward acceleration of magnitude 0.80 m/s^2. What is the rotational inertia of the device about its axis of rotation?

Figure 8



A) 1.7 kg m^2
B) 2.6 kg m^2
C) 4.2 kg m^2
D) 5.0 kg m^2
E) 3.2 kg m^2

Ans:

For the block: $T = m(g + a)$

For the device: $\tau = FR - T_r = I_\alpha$

$$\Rightarrow I = \frac{r}{a}[Fr - m(g + a)r] = 1.7 \text{ kg m}^2$$

Q17.

A 0.50-kg object moves in a horizontal circular track with radius of 2.0 m. An external force of 4.0 N always tangent to the track causes the object to speed up as it goes around. What is the work done by the external force as the object makes one revolution?

A) 50 J
B) 60 J
C) 70 J
D) 80 J
E) 0 J

Ans:

$W = t\theta$

$\theta = 2\pi\text{rad}$

$\tau = Fr = 4 \times 2 = 8\ N \cdot m$

$W = 50.3\ \text{J}$

Q18.

A uniform solid sphere rolls smoothly without slipping with center of mass velocity of 2.5 m/s along a horizontal floor, then up a ramp inclined at 20°. What is the maximum distance traveled by the center of mass of the sphere along the ramp?

A) 1.3 m
B) 1.0 m
C) 1.7 m
D) 2.1 m
E) 2.4 m

Ans:

$$\frac{1}{2}\mathrm{m}V_{cm}^2 + \frac{1}{2}I\omega^2 = mgdsin\theta$$

$$\frac{1}{2}\mathrm{m}v_{cm}^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\frac{V^2}{R^2} = mgdsin\theta$$

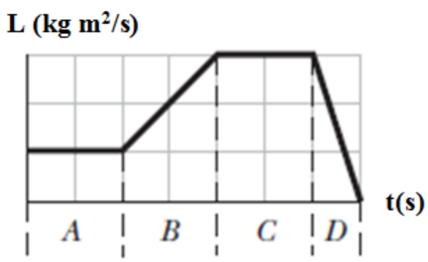
$$d = \frac{\frac{7}{10}V_{cm}^2}{gsin\theta} = 1.3\ m$$

Q19.

Figure 9 is a plot of the angular momentum of a wheel versus time. At which time interval (A, B, C or D) is the magnitude of the torque acting on the wheel maximum?

A) D only
B) A only
C) B only
D) C only
E) A and C

Figure 9



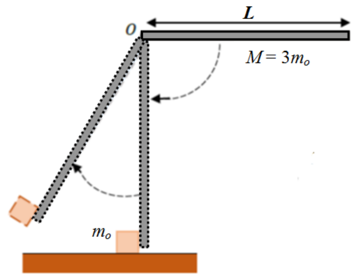
Ans:

$$\tau = \frac{\Delta L}{\Delta t} = slope$$

Q20.

The uniform thin rod in **Figure 10** (mass $M = 3m_o$ and length $L = 1.5$ m) can rotate about a vertical axis through one end (O). Initially, it is held horizontally. When released, it swings through its lowest position and collides with a stationary block of mass m_o that sticks to the end of the rod. What is the angular speed (in rad/s) of the rod-block system immediately after the collision? Take $m_o = 0.50$ kg. (Ignore all forms of frictions)

Figure 10



A) 2.2
B) 3.2
C) 1.5
D) 3.6
E) 4.0

Ans:

1) Conservation of Energy:

$$\mathrm{Mg}\,\frac{L}{2} = \frac{1}{2} I_R \omega_i^2 \;\left(I_R = \frac{1}{L} ML^2 = m_0 L^2\right)$$

$$\Rightarrow \omega_i = \sqrt{\frac{3g}{L}}$$

2) Conservation of Angular Momentum:

$$\mathrm{I}_f \omega_f = \mathrm{I}_i \omega_i \Rightarrow \omega_f = \frac{\mathrm{I}_i \omega_i}{\mathrm{I}_f}$$

$$\mathrm{I}_i = \frac{M}{3} L^2 = m_0 L^2, \mathrm{I}_f = \frac{M}{3} L^2 + m_0 L^2 = 2m_0 L^2$$

$$\omega_f = \frac{m_0 L^2 \omega_i}{2m_0 L^2} = \frac{\omega_i}{2} = \frac{1}{2}\sqrt{\frac{3g}{L}} = 2.2\ rad/s$$