Q1.

Assume the equation $x = At^3 + Bt$ describes the motion of a particular object, with x having the dimension of length and t having the dimension of time. Determine the dimensions of the constants A and B respectively.

A) M^0L/T^3, M^0L/T
B) M^0L/T^2, M^0L/T^2
C) ML/T^3, ML/T
D) M^0L/T, M^0L/T
E) ML/T, ML/T

Ans:

$$[A] = \left[\frac{x}{t^3}\right] = M^0\, L/T^3\,; [B] = \left[\frac{x}{t}\right] M^0\, L/T$$

Q2.

Gold, which has a density of 19.32 g/cm^3, can be pressed into a thin leaf. If gold with a mass of 27.63 g, is pressed into a leaf of 1.000 μm thickness, what is the area of the leaf?

A) 1.430 m^2
B) 0.545 m^2
C) 1.115 m^2
D) 0.755 m^2
E) 1.945 m^2

Ans:

$$m = \rho \times V = \rho \times (A \times t) \Longrightarrow A = \frac{m}{\rho \times t}$$

$$m = 27.63\ \ g = 27.63 \times 10^{-3} kg$$

$$\rho = 19.32\ \ g/cm^3 = 19.32 \times 10^3\ \ kg/m^3$$

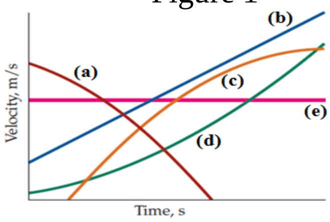
$$t = 10^{-6}\, m$$

$$A = \frac{27.63 \times 10^{-3}}{19.32 \times 10^3 \times 10^{-6}} = \frac{27.63}{19.32} = 1.430\, m^2$$

Q3.

Which one of the velocity-time graphs shown in **Figure 1** best describes the motion of an object with positive increasing acceleration?

Figure 1



A) d
B) a
C) b
D) c
E) e

Ans:

A

Q4.

The velocity of a particle moving along the x-axis is given, for $t > 0$, by : $v_x = (50t - 2.0t^3)$ m/s, where t is in s. What is acceleration of the particle (after t = 0) when it achieves its maximum displacement in the positive x-direction?

A) -1.0×10^2 m/s^2
B) -1.9×10^2 m/s^2
C) $+1.9\times10^2$ m/s^2
D) $+1.0\times10^2$ m/s^2
E) -1.5×10^2 m/s^2

Ans:

$$v_x = 50\,t - 2.0\,t^3 \; and \; a_x = 50 - 6t^2$$

$$For \max displacement, v_x = 0 \; but \; v_x = 0 = 50\,t - 2t^3\,; \;\; 50 - 2t^2 = 0\,; t = \pm 5\,s$$

$$t = \pm 5; \;\; a_x = 50 - 6t^2 = 50 - 150 = -100 \;\, m/s^2$$

Q5.

An object is thrown vertically upward. It has an upward velocity of 25 m/s when it reaches one fourth of its maximum height above its launch point. What is initial launch speed of the object? (Ignore the air resistance)

A) 29 m/s
B) 12 m/s
C) 17 m/s
D) 33 m/s
E) 35 m/s

Ans:

$$V_f^2 = V_1^2 - 2g\left(\frac{h}{4}\right) = V_i^2 - \frac{g}{2}\,h \;\; but \;\; h = \frac{V_i^2}{2g}$$

$$V_f^2 = V_i^2 - \frac{g}{2} \times \frac{V_i^2}{2g} = V_i^2 - \frac{V_i^2}{4} = \frac{3V_i^2}{4}$$

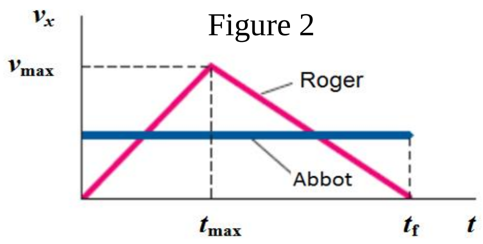
$$V_i = \sqrt{\frac{4V_f^2}{3}} = \sqrt{\frac{4}{3} \times (25)^2} = 28.9 \;\, m/s$$

Q6.

Two runners Roger and Abbot are running on a straight track along *x* axis from $t = 0$ to $t = t_f$. Abbot runs throughout the interval $t = 0$ to $t = t_f$ at a constant speed v_{Abbot}, while Roger has a velocity that depends upon time as shown in **Figure 2** ($t_{max} \neq t_f/2$). Both the runners travelled the same displacement during the time interval. What is the relation between v_{Abbot} and v_{max}.

A) $v_{Abbot} = v_{max}/2$
B) $v_{Abbot} = v_{max}$
C) $v_{Abbot} = 2\, v_{max}$
D) $v_{Abbot} = 3\, v_{max}/2$
E) $v_{Abbot} = v_{max}/3$

Figure 2



Ans:

$$\Delta X_{Roger} = \frac{1}{2} t_{max} V_{max} + \frac{1}{2}(t_f - t_{max}) V_{max} = \frac{1}{2} t_f V_{max}$$

$$\Delta X_{Roger} = t_f \times V_{Abbot}$$

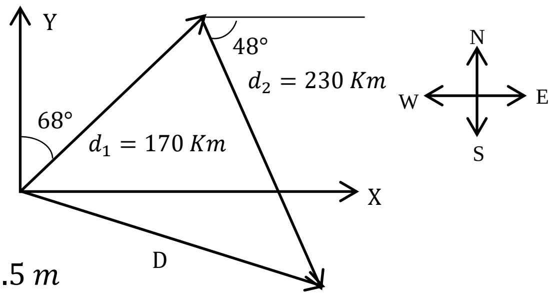
$$t_f \cdot V_{Abbot} = \frac{1}{2} V_{max} \cdot t_f$$

$$V_{Abbot} = \frac{V_{max}}{2}$$

Q7.

A plane leaves the airport and flies 68.0° east of north and travels 170 km. Then it changes direction of motion to fly 48.0° south of east, and travels 230 km. Finally, it makes an immediate emergency landing on the ground. When the airport sends out a rescue team, how far the team flies to go directly to this plane?

A) 329 km
B) 225 km
C) 250 km
D) 379 km
E) 401 km



Ans:

$$D_x = d_1 cos(22) + d_2 cos(48) = 311.5\ m$$

$$D_y = d_1 sin(22) - d_2 sin(48) = -107.2\ m$$

$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(311.5)^2 + (-107.2)^2} = 329.4\ km$$

Q8.

Vector $\vec{A}$, which is directed along an x axis, is to be added to vector $\vec{B}$ with a magnitude of 9.00 m. The sum is a third vector that is directed along the y axis, with a magnitude that is 5.00 times that of $\vec{A}$. What is the magnitude of $\vec{A}$?

A) 1.77 m
B) 0.54 m
C) 1.11 m
D) 2.54 m
E) 3.96 m

Ans:

$$\vec{C} = |A|\hat{\imath} + \vec{B} = 5|A|\hat{\jmath}$$

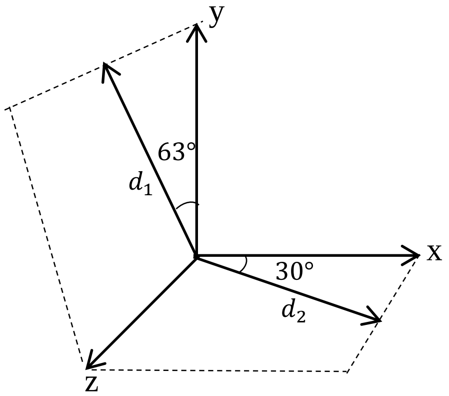
$$\vec{B} = 5|A|\hat{\jmath} - |A|\hat{\imath}$$

$$|B| = \sqrt{(5|A|)^2 + |A|^2} = 9$$

$$|A| = \frac{9}{\sqrt{26}} = 1.77$$

Q9.

Displacement $\vec{d}_1$ and $\vec{d}_2$ are in the yz and the xz planes, respectively. The displacement $\vec{d}_1$, with a magnitude of 4.50 m, makes an angle of 63.0° with the positive direction of the y axis. The displacement $\vec{d}_2$, with a magnitude of 1.40 m, makes an angle of 30.0° with the positive direction of the x axis. Both $\vec{d}_1$ and $\vec{d}_2$ have positive z components. What is the vector $\vec{d}_1 \times \vec{d}_2$?



A) $1.43\hat{i} + 4.86\hat{j} - 2.48\hat{k}$
B) $4.86\hat{i} + 2.48\hat{j} - 1.43\hat{k}$
C) $2.48\hat{i} - 1.43\hat{j} - 4.86\hat{k}$
D) $3.43\hat{i} + 2.77\hat{j} + 1.45\hat{k}$
E) $2.22\hat{i} + 3.66\hat{j} - 3.33\hat{k}$

Ans:

$$d_1 = 4.5\cos 63\,\hat{\jmath} + 4.5\sin 63\,\hat{k} = 2.04\,\hat{\jmath} + 4.0\,\hat{k}$$

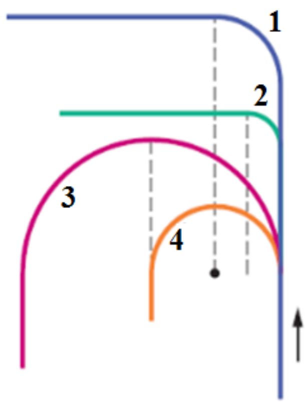
$$d_2 = 1.4\cos 30\,\hat{\imath} + 1.4\sin 30\,\hat{k} = 1.21\,\hat{\imath} + 0.7\,\hat{k}$$

$$\vec{d}_1 \times \vec{d}_2 = (2.04\,\hat{\jmath} + 4.0\,\hat{k}) \times (1.21\,\hat{\imath} + 0.7\,\hat{k}) = 1.43\,\hat{\imath} + 4.86\,\hat{\jmath} - 2.48\,\hat{k}$$

Q10.

Figure 3 shows four tracks (either half- or quarter-circles) that can be taken by a train, which moves at a constant speed. The radii of curvature of the tracks are shown by dash lines. Rank the tracks according to the magnitude of a train's acceleration on the curved portion, greatest first.

Figure 3



A) 2, then 1 and 4 tie, then 3
B) 1, then 2 and 4 tie, then 3
C) 3, then 1 and 2 tie, then 4
D) 4, then 3 and 1 tie, then 2
E) 4, then 1 and 2 tie, then 3

Ans:

$$a_R = \frac{v^2}{R} \Longrightarrow a_R \propto \frac{1}{R}$$

$$R_2 < (R_1, R_4) < R_3$$

Q11.

A fish swimming in a horizontal plane has an initial velocity $\vec{v}_i = (4.0\,\hat{i} + 1.0\,\hat{j})$ m/s at a point in the ocean. After the fish swam for 20 s, its velocity is $\vec{v} = (20\,\hat{i} - 5.0\,\hat{j})$ m/s. If the fish maintains constant acceleration, what is its velocity in (m/s) at t = 25 s.

A) $24\,\hat{i} - 6.5\,\hat{j}$
B) $16\,\hat{i} + 9.5\,\hat{j}$
C) $11\,\hat{i} - 8.8\,\hat{j}$
D) $33\,\hat{i} + 17\,\hat{j}$
E) $27\,\hat{i} - 16\,\hat{j}$

Ans:

$$\vec{v}_f = \vec{v}_i + \vec{a}\,t = but\ \vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t} = \frac{(20\,\hat{\imath} - 5\,\hat{\jmath}) - (4\,\hat{\imath} + 1.0\,\hat{\jmath})}{20} = 0.8\,\hat{\imath} - 0.3\,\hat{\jmath}$$

After t = 25 s

$$v_f = (4\,\hat{\imath} + 10\,\hat{\jmath}) + 25(0.8\,\hat{\imath} - 0.3\,\hat{\jmath}) = 24\,\hat{\imath} - 6.5\,\hat{\jmath}$$

Q12.

A stone thrown horizontally from the top of a 24.0 m high tower hits the ground at a point 18.0 m from the base of the tower. Find the speed of the stone just before it hits the ground. (Ignore any effects due to air resistance).

A) 23.2 m/s
B) 18.9 m/s
C) 25.5 m/s
D) 14.3 m/s
E) 29.9 m/s

Ans:

$$For\ motion\ along\ y-axis\ \ time\ t = \sqrt{\frac{2\times 24}{9.8}} = 2.213\ s$$

$$v_x = \frac{18}{2.213} = 8.134\ \ m/s$$

$$v_y = -9.8 \times 2.213 = 21.69\ \ \ m/s$$

$$|v| = \sqrt{v_x^2 + v_y^2} = \sqrt{(8.31)^2 + (21.69)^2} = 23.22\ m/s$$

Q13.

A boat takes 3.0 hours to travel 30 km downstream a river, then it takes 5.0 hours to return (upstream the river). How fast is the river flowing?

A) 2.0 km/h
B) 1.5 km/h
C) 1.1 km/h
D) 2.5 km/h
E) 2.9 km/h

Ans:

$$For\ downstream\ motion\ 30 = (v_{boat} + v_{river}) \times 3\ \Rightarrow v_{boat} + v_{river} = 10\ \longrightarrow (1)$$

$$For\ upstream\ \ motion\ 30 = (v_{boat} - v_{river}) \times 5\ \Rightarrow v_{boat} - v_{river} = 6\ \longrightarrow (2)$$

Solving (1) and (2)

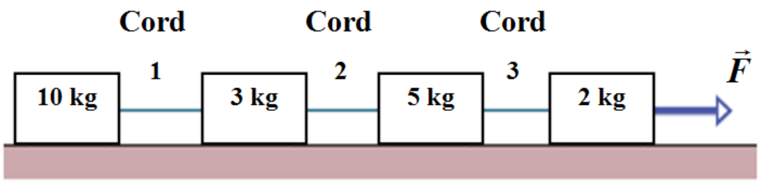
$$2v_{river} = 4 \Rightarrow v_{river} = 2$$

Q14.

Figure 4 shows four blocks connected by cords, being pulled across a frictionless floor by force $\vec{F}$. Rank the cords according to their tension, greatest first.

A) 3, 2 ,1
B) 2, 1, 3
C) 1, 2, 3
D) 3, 1, 2
E) 2, 3 ,1

Figure 4



Ans:

$$a = \frac{F}{\sum m} = \frac{F}{20}$$

$$T_1 = 10 \times a = 0.5\,F$$

$$T_2 = 13 \times a = 0.65\,F$$

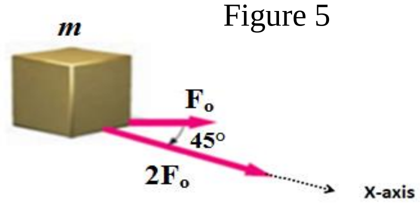
$$T_3 = 18 \times a = 0.9\,F$$

Q15.

A horizontal force of magnitude F_o causes an acceleration of magnitude 3.0 m/s^2 when it acts on an object of mass *m* sliding on a frictionless surface. Find the acceleration of the same object when two forces F_o and $2F_o$, as shown in **Figure 5**, act on it.

A) $(8.1\,\hat{i} + 2.1\,\hat{j})$ m/s^2
B) $(2.1\,\hat{i} - 2.1\,\hat{j})$ m/s^2
C) $(6.5\,\hat{i} + 3.3\,\hat{j})$ m/s^2
D) $(8.1\,\hat{i} - 2.1\,\hat{j})$ m/s^2
E) $(6.5\,\hat{i} - 3.3\,\hat{j})$ m/s^2

Figure 5



Ans:

$$\vec{a} = \frac{\vec{F}_0 + 2\vec{F}_0}{m}; m = \frac{F_0}{a} = \frac{F_0}{3r}$$

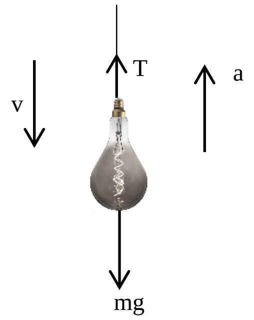
$$= \frac{2F_0\hat{\imath} + F_0\cos 45\,\hat{\imath} + F_0 \sin 45\,\hat{\jmath}}{F_0/3} = 3(2\hat{\imath} + \cos 45\hat{\imath} + \sin 45\,\hat{\jmath})$$

$$a = 3(2.71\,\hat{\imath} + 0.71\,\hat{\jmath}) = 8.1\hat{\imath} + 2.1\,\hat{\jmath}$$

Q16.

A lamp hangs vertically from a cord in a descending elevator that decelerates at 2.4 m/s^2. If the tension in the cord is 89 N, what is the lamp's mass?

A) 7.3 kg
B) 5.1 kg
C) 3.2 kg
D) 8.1 kg
E) 8.9 kg



Ans:

$$T - mg = ma$$

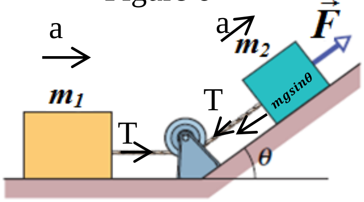
$$m = \frac{T}{g+a} = \frac{89}{9.8 + 2.4} = 7.3\ kg$$

Q17.

In **Figure 6**, a force $\vec{F}$ of magnitude 12 N is applied to a box of mass m_2 = 1.0 kg. The force is directed up a plane tilted by θ = 37°. The box is connected by a cord to a second box of mass m_1= 3.0 kg on the floor. The floor, plane, and pulley are frictionless, and the masses of the pulley and cord are negligible. What is the tension in the cord?

A) 4.6 N
B) 6.4 N
C) 2.2 N
D) 1.3 N
E) 5.5 N

Figure 6



Ans:

$$For\ m_1;\quad T = m_1 a = 3a \Rightarrow a = \frac{T}{3}$$

$$For\ m_2;\quad F - m_2 g sin\theta - T = m_2 a = a\ (m = 1.0\ kg)$$

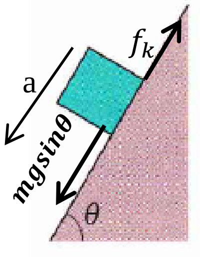
$$12 - 9.8 \times \sin 37 - T = \frac{T}{3}$$

$$T = \frac{3 \times 6.10}{4} = 4.58\ N$$

Q18.

As shown in **Figure 7**, the coefficient of kinetic friction between the block and inclined plane is 0.200, and angle θ is 60.0°.What is the magnitude and direction (upward or downward the plane) of the blocks acceleration.

Figure 7



A) 7.51 m/s^2, downward
B) 7.51 m/s^2, upward
C) 3.59 m/s^2, downward
D) 5.49 m/s^2, downward
E) 8.82 m/s^2, upward

Ans:

$$f_k = \mu_k mgcos\theta = 0.2 \times m \times 9.8 \times 0.50 \times 0.98\, m$$

$$mgsin\theta = m \times 9.8 \times sin\theta = m \times 9.8 \times 0.87 = 8.487$$

$$since\ mgsin\theta > f, downward\ motion, also\ accelaration\ a\ downward$$

$$mgsin\theta - f_k = ma$$

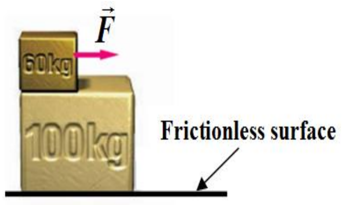
$$8.47 \times m - 0.98\, m = ma$$

$$a = 8.487 - 0.980 = 7.507\ m/s^2$$

Q19.

A 60 kg block slides along the top of a 100 kg block. The 60 kg block has an acceleration of 3.0 m/s^2 while a horizontal force F of 320 N is applied to it, as shown in **Figure 8**. There is friction between the two blocks. Find the magnitude of the acceleration of the 100 kg block during the time that the 60 kg block remains in contact with it. (The surface under 100 kg block is frictionless).

Figure 8



A) 1.4 m/s^2
B) 2.1 m/s^2
C) 0.5 m/s^2
D) 0.8 m/s^2
E) 2.8 m/s^2

Ans:

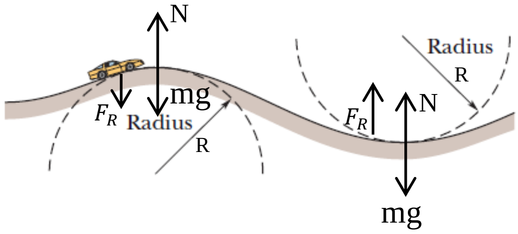
$$For\ 60\ kg\ block\ F - f = ma \Rightarrow f = F - ma = 320 - 60 \times 3 = 140\ N$$

$$For\ 100\ kg\ block\ f = m'a' \Rightarrow a' = \frac{f}{m'} = \frac{140}{100} = 1.40\ m/s^2$$

Q20.

In **Figure 9**, a car is driven at constant speed over a circular hill and then into a circular valley with the same radius. At the top of the hill, the normal force on the driver from the car seat is zero. The driver's mass is 80.0 kg. What is the magnitude of the normal force on the driver from the seat when the car passes through the bottom of the valley?

Figure 9



A) 1.57×10^3 N
B) 1.01×10^3 N
C) 1.17×10^3 N
D) 2.81×10^3 N
E) 2.22×10^3 N

Ans:

$At\ the\ top\ N - mg = -F_R$

$But\ N = 0, F_R = mg$

In the Valley:

$N - mg = F_R$

$N = F_R + mg = mg + mg = 2mg$

$= 2 \times 80 \times 9.8 = 1568\ N = 1.57 \times 10^3$ N