Q1.

The magnitude of a force applied on an object is given by the relation: $F = k\rho^x v^y t^z$, where *k* is dimensionless constant, *ρ* is density, *v* is speed and *t* is time. The values of *x*, *y* and *z;* respectively; are:

A) 1, 4 and 2
B) 1, 2 and 3
C) 2, 1 and 2
D) 1, 2 and 4
E) 1, 4 and 3

Solution

$$\vec{F} = kP^x v^y t^z$$

$$MLT^{-2} = (ML^{-3})^x (LT^{-1})^y T^z$$

$$MLT^{-2} = M^x L^{-3x+y} T^{-y+z}$$

$$x = 1$$

$$-3x + y = 1 \qquad \Rightarrow y = 4$$

$$-y + z = -2 \qquad \Rightarrow z = 2$$

Q2.

A car covers $\frac{2}{5}$ of the total distance at the speed of 45.0 km/h and the remaining $\frac{3}{5}$ of the total distance at the speed of 63.0 km/h. The average speed of the car during the whole trip is:

A) 15.1 m/s
B) 24.3 m/s
C) 31.1 m/s
D) 10.5 m/s
E) 45.3 m/s

Solution

$$t_1 = \frac{2}{5}\frac{x}{v_1} \; ; \; t_2 = \frac{3}{5}\frac{x}{v_2}$$

$$v_{av} = \frac{x}{t_1 + t_2} = \frac{x}{\frac{2x}{5v_1} + \frac{3x}{5v_2}} = \frac{5xv_1v_2}{x(2v_2 + 3v_1)} = \frac{5v_1v_2}{(2v_2 + 3v_1)}$$

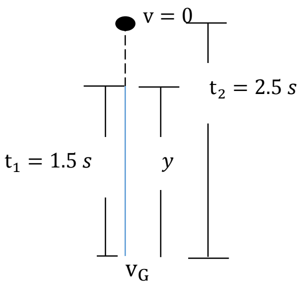
$$v_1 = \frac{45 \times 1000}{3600}\frac{m}{s} = 12.5\, m/s\, ; \; v_2 = \frac{63 \times 1000}{3600}\frac{m}{s} = 17.5\, m/s$$

$$\therefore v_{av} = \frac{5 \times 12.5 \times 17.5}{(2 \times 17.5 + 3 \times 12.5)} = 15.1 \;\, m/s$$

Q3.

A rock is thrown vertically upward from ground level at time t = 0. At t = 1.5 s it passes the top of a tall tower, and 1.0 s later it reaches its maximum height. What is the height of the tower? (Ignore air resistance)

A) 25.7 m
B) 14.4 m
C) 37.1 m
D) 45.5 m
E) 57.4 m



Solution

$$v = v_0 + at_2$$

$$0 = v_0 - 9.8 \times 2.5$$

$$v_0 = 24.5\ m/s$$

$$y = v_0 t_1 + \frac{1}{2} a t_1^2 = 24.5 \times 1.5 - \frac{1}{2} \times 9.8 \times (1.5)^2 = 25.7\ m$$

Q4.

Figure 1 shows the displacement - time graph of a particle moving along the x-axis. Which one of the following statements about the particle's motion is **FALSE**?

A) The particle changes its direction at 4 s.
B) Over the whole trip, the magnitude of displacement of the particle is 3 times smaller than its distance.
C) The particle's speeds for the first 2s (t =0 to t =2s) and next 2s (t = 2s to t = 4s) are equal.
D) The particle changes its direction at 2 s.
E) The particle moves forward (along the positive x-direction), reverses direction and moves beyond the starting point.

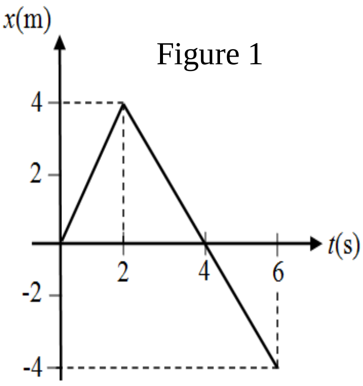


Figure 1

Ans:

A

Q5.

If the position of a particle is given by $x(t) = 10t - 5t^2$, where x is in meters and t is in seconds, find the magnitude of average velocity of the particle from t = 0 to the time for maximum position along the positive x-direction.

A) 5 m/ s
B) 10 m/s
C) 20 m/s
D) 15 m/s
E) 2 m/s

Solution

$$x(t) = 10t - 5t^2$$

$$v(t) = \frac{dx(t)}{dt} = 10 - 10\,t$$

For maximum positive x-position

$$v(t) = 0 \implies t = 1\text{ s}$$

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x(1) - x(0)}{1} = \frac{5 - 0}{1} = 5\text{ m/s}$$

Q6.

Consider two non-zero vectors $\vec{A}$ and $\vec{B}$. If $|\vec{A} + \vec{B}|^2 = |\vec{A}|^2 + |\vec{B}|^2$ then,

A) $\vec{A}$ and $\vec{B}$ must be perpendicular to each other.
B) $\vec{A}$ and $\vec{B}$ must be parallel to each other.
C) The angle between $\vec{A}$ and $\vec{B}$ must be 180°.
D) The angle between $\vec{A}$ and $\vec{B}$ must be 45°.
E) The angle between $\vec{A}$ and $\vec{B}$ must be 30°.

Solution

$$|\vec{A} + \vec{B}|^2 = (\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B}) = |\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta$$

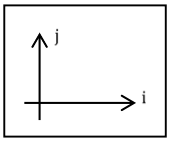
For $\theta = 90°$

$$|\vec{A} + \vec{B}|^2 = |\vec{A}|^2 + |\vec{B}|^2$$

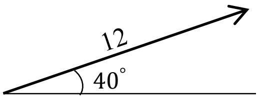
Q7.

In the sum $\vec{A}+\vec{B}=\vec{C}$, vector $\vec{A}$ has a magnitude of 12.0 units and is directed at 40.0° counterclockwise from the +*x* direction. Vector $\vec{C}$ has a magnitude of 15.0 units and is directed at 20.0° clockwise from the –*x* direction. Find the vector $\vec{B}$.

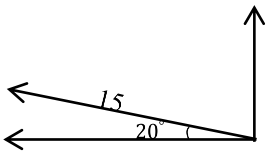
A) $-23.3\hat{i}-2.60\hat{j}$
B) $1.35\hat{i}+7.15\hat{j}$
C) $17.5\hat{i}+15.8\hat{j}$
D) $23.5\hat{i}+11.9\hat{j}$
E) $-17.5\hat{i}+15.8\hat{j}$



Solution



$$\vec{A} = 12\cos 40\hat{\imath} + 12\, sin\theta\hat{\jmath}$$

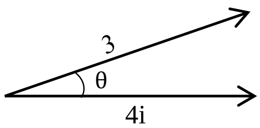


$$\vec{C} = -(15\cos 20° + 12\, cos40°)i + (15sin20° - 12sin40\,) = -23.3\hat{\imath} - 2.6\hat{\jmath}$$

Q8.

Consider two displacements, $\vec{d}_1$ with magnitude 3 m and $\vec{d}_2$ with magnitude 4 m. What angle does $\vec{d}_2$ make with $\vec{d}_1$ to get a resultant displacement of magnitude 4 m?

A) 112°
B) 25°
C) Zero
D) 156°
E) 90°



Solution

$$\vec{r} = (4 + 3cos\theta)\hat{\imath} + 3sin\theta\hat{\jmath}$$

$$|r|^2 = (x)^2 + (y)^2$$

$$|r|^2 = (4 + 3cos\theta)^2 + 9\, sin^2\theta$$

$$16 = 16 + 24cos\theta + 9$$

$$\theta = cos^{-1}\left(-\frac{9}{24}\right) = 112°$$

Q9.

Here are three vectors:

$\vec{d}_1 = 3\hat{i} + 2\hat{k}$

$\vec{d}_2 = 2\hat{i} + 4\hat{j}$

$\vec{d}_3 = 3\hat{j} + \hat{k}$

The result of $\vec{d}_1 . (\vec{d}_2 \times \vec{d}_3)$ is:

A) 24
B) 37
C) 65
D) 11
E) Zero

Solution

$\vec{d}_1 = 3\hat{\imath} + 2\hat{k}$

$\vec{d}_2 = 2\hat{\imath} + 4\hat{\jmath}$

$\vec{d}_3 = 3\hat{\jmath} + \hat{k}$

$\vec{d}_2 \times \vec{d}_3 = (2\hat{\imath} + 4\hat{\jmath}) \times (3\hat{\jmath} + \hat{k}) = 6\hat{k} + 2(-\hat{\jmath}) + 4\hat{\imath} = 4\hat{\imath} - 2\hat{\jmath} + 6\hat{k}$

$\vec{d}_1 \cdot \vec{d}_2 \times \vec{d}_3 = (3\hat{\imath} + 2\hat{k}) \cdot (4\hat{\imath} - 2\hat{\jmath} + 6\hat{k}) = 12 + 12 = 24$

Q10.

A particle P travels with constant speed on a circle of radius r = 3.00 m as shown in **Figure 2** and completes one revolution in 20.0 s. The particle passes through O at time t = 0. With respect to O, find the particle's position vector and acceleration at the time t = 15 s.

A) $-3.00\text{ m }\hat{\text{i}} + 3.00\text{ m }\hat{\text{j}}$ and $0.296\ \hat{\text{i}}\ \text{m/s}^2$

B) $1.35\text{ m }\hat{\text{i}} + 7.15\text{ m }\hat{\text{j}}$ and $3.00\ \hat{\text{i}}\ \text{m/s}^2$

C) $7.25\text{ m }\hat{\text{i}} + 5.18\text{ m }\hat{\text{j}}$ and $-3.00\ \hat{\text{i}}\ \text{m/s}^2$

D) $-3.00\text{ m }\hat{\text{i}} + 3.00\text{ m }\hat{\text{j}}$ and $0.532\ \hat{\text{i}}\ \text{m/s}^2$

E) $-1.35\text{ m }\hat{\text{i}} + 7.15\text{ m }\hat{\text{j}}$ and $-0.532\ \hat{\text{j}}\ \text{m/s}^2$

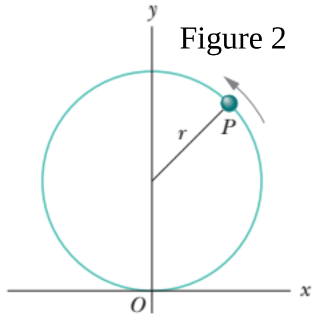
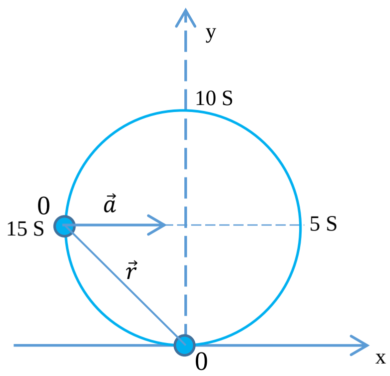


Figure 2

Solution



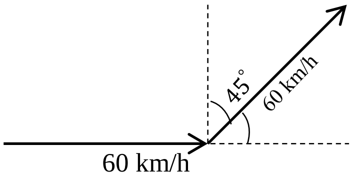
$$\vec{r} = -3\hat{\imath} + 3\hat{\jmath}$$

$$\vec{a} = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{20}\right)^2}{r} = \frac{\pi^2 r}{100} = \frac{3.14^2 \times 3}{100} = 0.296\,\hat{\imath}$$

Q11.

A train moves at a constant speed of 60.0 km/h due east for 40.0 min, and then moves with the same speed in the direction 45.0° east of north for 20.0 min. What are the magnitude and direction of its average velocity during this trip?

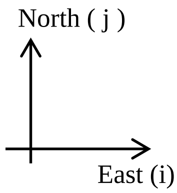
A) 56.0 km/h, 14.6° North of East
B) 28.3 km/h, 45.0° North of East
C) 10.0 km/h, 23.0° South of East
D) 28.3 km/h, 86.2° South of East
E) 56.0 km/h, 36.5° North of East



Solution

$$x_1 = \frac{60}{60} \times 40 = 40\ km\ \hat{\imath}$$

$$x_2 = \frac{60}{60} \times 20\ sin45^{\circ}\ \hat{\imath}$$



$$y_2 = \frac{60}{60} \times 20\ cos45^{\circ}\ \hat{\jmath}$$

$$x = (40 + 20\ cos45^{\circ}\ ) = 54.1\ km\ \hat{\imath}$$

$$y = 20\ cos\ 45^{\circ}\ \ \hat{\jmath} = 14.1\ \hat{\jmath}$$

$$t = 40 + 20 = 60\ min = 1\ hr$$

$$v_{xav} = 54.1\ km/h\ \ \hat{\imath}$$

$$v_{yav} = 14.1\ km/h\ \ \hat{\jmath}$$

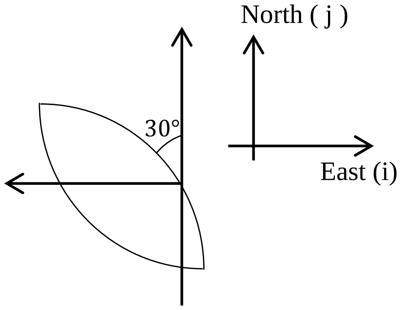
$$|v_{av}| = \sqrt{(54.1)^2 + (14.1)^2} = 56\ km/h$$

$$\theta = tan^{-1}\left(\frac{14.1}{54.1}\right) = 14.6°\ North\ of\ East$$

Q12.

A 250-m wide river flows due east at a uniform speed of 2.0 m/s. A boat with a speed of 8.0 m/s relative to the water leaves the south bank pointed in a direction 30° west of north. How long does the boat take to cross the river?

A) 36 s
B) 57 s
C) 24 s
D) 13 s
E) 85 s



Solution

$v_{BR} = -8\,sin30\,\hat{\imath} + 8cos30\,\hat{\jmath}$

$v_{\text{REx}} = 2\hat{\imath}, \qquad v_{\text{REy}} = 0$

Since the boat is moving towards north (ĵ) with respect to ground (earth: E), only ĵ motion is important.

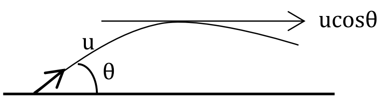
$v_{BRy} = v_{BEy} - \cancel{v_{REy}}^{\,0}$

$v_{BEy} = 8cos30°$

$$t = \frac{y}{v_{BEy}} = \frac{250}{4\sqrt{3}} = 36s$$

Q13.

A projectile is fired with an initial speed u at an angle θ above the horizontal. What is the change in speed when it reaches the highest point? (Ignore air resistance.)

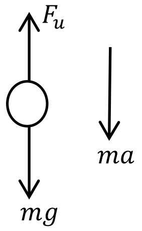


A) $(u\cos\theta - u)$
B) $u\cos\theta$
C) u
D) $u\sin\theta$
E) $(u\cos\theta + u)$

Q14.

A hot-air balloon of mass 100 kg is descending vertically with downward acceleration of magnitude 3.00 m/s^2. How much mass must be thrown out of the balloon to give the balloon an upward acceleration of magnitude 3.00 m/s^2. Assume that the upward force from the air (the lift) does not change because of the decrease in mass.

A) 46.9 kg
B) 90.1 kg
C) 25.7 kg
D) 67.4 kg
E) 13.5 kg

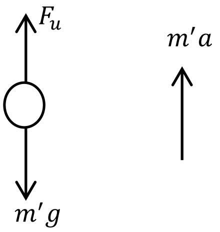


Solution

$$F_u - mg = -ma$$

$$F_u = mg - ma \longrightarrow (1)$$

Next



$$F_u - m'g = m'a$$

$$F_u = m'g + m'a \longrightarrow (2)$$

From equations (1) and (2)

$$m' = \frac{m(g-a)}{(a+g)}$$

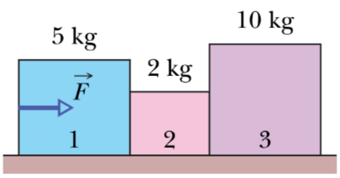
$$m' = \frac{100 \times 6.8}{12.8} = 53.1\ kg$$

$$\Delta m = m - m' = 46.9\ kg$$

Q15.

Figure 3 shows three blocks being pushed across a frictionless floor by horizontal force $\vec{F}$. Rank the force $\vec{F}$, $\vec{F}_{21}$ (force on block 2 from block 1) and $\vec{F}_{32}$ (force on block 3 from block 2) according to magnitude, **greatest first**.

Figure 3



A) $\vec{F}, \vec{F}_{21}, \vec{F}_{32}$
B) $\vec{F}_{21}, \vec{F}_{32}, \vec{F}$
C) $\vec{F}$, then ($\vec{F}_{21}$ and $\vec{F}_{32}$) tie
D) $\vec{F}_{32}, \vec{F}, \vec{F}_{21}$
E) All tie

Solution

$|F| = 17\ a$

$|F_{21}| = 12\ a$

$|F_{32}| = 10\ a$

$|F| > F_{21} > F_{32}$

Q16.

Figure 4 shows two blocks connected by a cord (of negligible mass) that passes over a frictionless pulley (also of negligible mass). One block has mass m_1 = 65.0 kg; the other block, which is at the height h = 10.0 m from the ground, has mass m_2 = 85.0 kg. If the system is released from rest with what speed does m_2 hit the ground? (Ignore air resistance)

A) 5.11 m/s
B) 1.07 m/s
C) 3.70 m/s
D) 2.32 m/s
E) 4.05 m/s

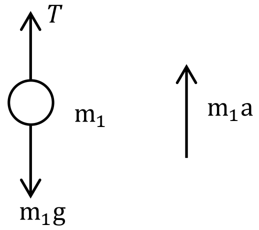
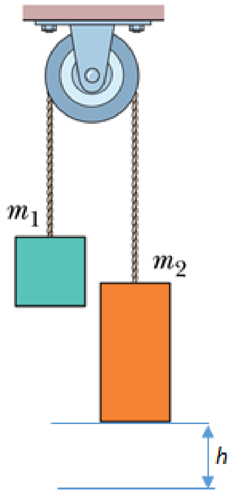


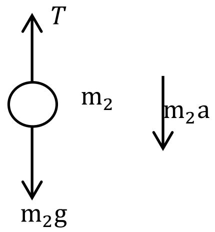
Figure 4



Solution

$$T - m_1 g = m_1 a$$

$$T = m_1 g + m_1 a - - - - (1)$$



$$T = m_2 g - m_2 a - - - - (2)$$

$$m_2 g + m_1 a = m_2 g - m_2 a$$

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2} = 1.3\ m/s^2$$

$$v_0 = 0,\ \ \Delta y = -10\ m,\ \ a = -1.3\ m/s^2$$

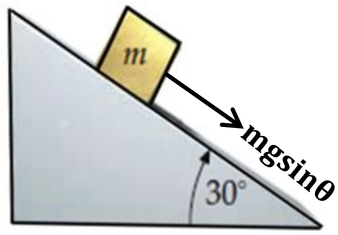
$$v = \sqrt{2 \times (-10) \times (-1.3)} = 5.11 m/s$$

Q17.

A block of mass m is at rest on a plane that is inclined at an angle of 30° with the horizontal, as shown in **Figure 5**. Which of the following relations about the magnitude of static friction force f_s is necessarily **TRUE**? (Ignore air resistance)

A) $f_s = mg\sin 30^o$
B) $f_s > mg\sin 30^o$
C) $f_s > mg$
D) $f_s > mg\cos 30^o$
E) $f_s = mg\cos 30^o$

Figure 5



Solution

Not moving: Static Friction

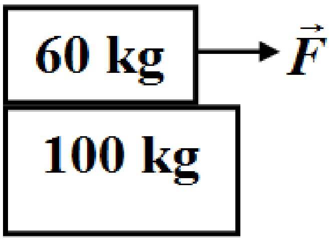
$$\therefore f_s = \text{mgsin}\theta$$

Q18.

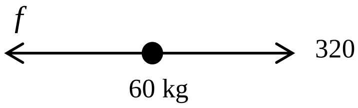
A 60 kg block slides along the top of a 100 kg block as shown in **Figure 6**. The 60 kg block has an acceleration of 3.0 m/s^2 while a horizontal force $\vec{F}$ = 320 N is applied to it. There is no friction between the 100 kg block and the horizontal surface on which it stands but there is friction between the two blocks. Find the magnitude of the acceleration of the 100 kg block during the time the two blocks remain in contact.

A) 1.4 m/s^2
B) 2.0 m/s^2
C) 3.7 m/s^2
D) 1.6 m/s^2
E) 3.0 m/s^2

Figure 6

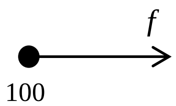


Solution



$$320 - f = 60\, a_1$$

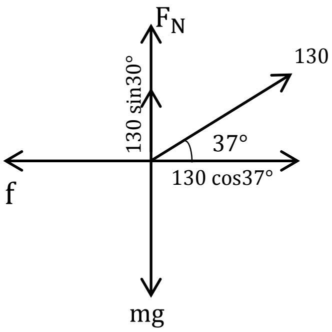
$$f = 320 - 60 \times 3 = 140\, N$$



$$f = 100 a_2 \Longrightarrow a_2 = \frac{f}{100} = \frac{140}{100} = 1.4\ \mathrm{m}/s^2$$

Q19.

A man is pulling his 20.0 kg suitcase at the constant speed of 0.500 m/s. He pulls it with 130 N of force at an angle of 37.0° above the horizontal as shown in **Figure 7**. The normal force on the suitcase and the force of friction on wheels of the suitcase are respectively:



A) 118 N and 104 N
B) 54.6 N and 86.4 N
C) 118 N and 86.4 N
D) 56.4 N and 104 N
E) 253 N and 140 N

Figure 7

Solution:

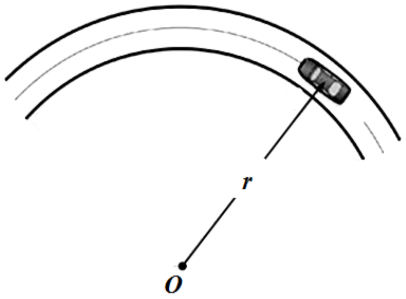
$$130\, sin30° + F_N = mg$$

$$F_N = 20 \times 9.8 - 130\, sin\, 30° = 118\, N$$

$$f = 130\, cos\, 37° = 104\, N$$

Q20.

When a car moves on a flat circular road at a constant speed of 15.0 m/s, the driver experiences a radial force of 120 N. What is the radial force on the driver if the speed of the car on this circular road is 30.0 m/s?



A) 480 N
B) 120 N
C) 224 N
D) 310 N
E) 548 N

Solution:

$$120 = \frac{m \times 15^2}{r} \ldots\ldots\ldots (1)$$

$$F = \frac{m \times 30^2}{r} \ldots\ldots\ldots (2)$$

Dividing equation (2) by (1) yields

F = 480 N