Q1.

Van der Wall's equation of state for gases is given by

$$(P + \frac{a}{V^2})(V - b) = RT$$

Where, P is the pressure (kg/m.s^2), V is the volume (m^3) and T is the temperature (K). a, b and R are constants. The dimension of "a" is

A) ML^5T^{-2}
B) L^2T^{-2}
C) L^6
D) $ML^{-1}T^{-2}$
E) ML^2T^{-2}

Ans:

$$\frac{[a]}{[v^2]} = [P]$$

$$[a] = ML^{-1}T^{-2}L^6 = ML^5T^{-2}$$

Q2.

Which ONE of the following statements is TRUE?

A) The instantaneous velocity of a particle is always directed along the tangent to the particle's path at the particle's position.
B) If the "velocity versus time" graph of an object is a horizontal line, that object is accelerating.
C) It is physically impossible for an object to have a negative acceleration and yet be speeding up.
D) Average speed is always less than the magnitude of average velocity.
E) In projectile motion, the vertical acceleration is zero at the maximum height.

Ans:

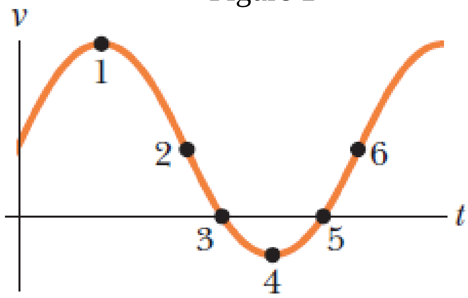
A

Q3.

Figure 1 gives the velocity as a function of time for a particle moving along an x- axis. Dot **1** is at the highest point on the curve, dot **4** is at the lowest point, and dots **2** and **6** are at the same height. At which point(s) does the particle change its direction?

A) 3 and 5
B) 1 and 4
C) 2 and 6
D) 3 only
E) 1 only

Figure 1



Ans:

A

Q4.

From t = 0 to t = 5.00 min, a man stands still, and from t = 5.00 min to t = 10.0 min, he walks in a straight line at a constant speed of 2.20 m/s. What is the average velocity v_{avg} in the time interval 3.00 min to 9.00 min?

A) 1.47 m/s
B) 2.20 m/s
C) 1.83 m/s
D) 3.67 m/s
E) 4.40 m/s

Ans:

$$\Delta t = 9 - 3 \text{ min} = 360 \text{ s}$$

$$\Delta t' = 9 - 5 \text{ min} = 240 \text{ s}$$

$$x_2 = V\Delta t' = 2.2 \times (240 \text{ s}) = 528 \text{ m}$$

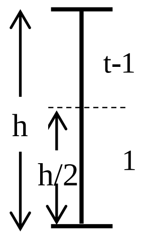
$$x_1 = 0$$

$$V_{av} = \frac{x_2 - x_1}{\Delta t} = \frac{528}{360} = 1.47 \text{ m/s}$$

Q5.

An object falls a distance h from rest. If it travels 0.50 h in the last 1.00 s, the height h of its fall is

A) 57.1 m
B) 32.6 m
C) 1.68 m
D) 85.1 m
E) 4.90 m



Ans:

$$h = \frac{1}{2} g t^2 \quad \rightarrow (1)$$

$$t = \sqrt{\frac{2h}{g}}$$

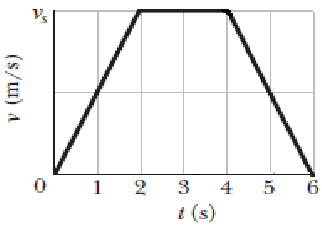
$$\frac{h}{2} = \frac{1}{2} g(t-1)^2 \quad \rightarrow (2)$$

$$\frac{h}{2} = \frac{1}{2} g \left(\sqrt{\frac{2h}{g}} - 1 \right)^2 \Rightarrow h = 57.1 \text{ m}$$

Q6.

A particle starts from the origin at t = 0 and moves along the positive x-axis. A graph of the velocity of the particle as a function of the time is shown in **Figure 2**; where the v-axis scale is set by v_s = 4.0 m/s. What is the average acceleration of the particle between t =1.0 s and t = 4.0 s?

Figure 2



A) 0.67 m/s²
B) 11 m/s²
C) 2.0 m/s²
D) zero
E) 6.0 m/s²

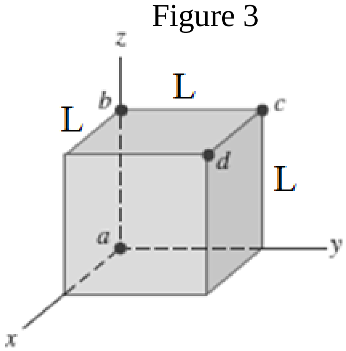
Ans:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{4-2}{4-1} = 0.67\ m/s^2$$

Q7.

A cube of edge length **L** is placed so that one corner is at the origin and three edges are along the x-, y-, and z-axes of a coordinate system (see **Figure 3**). What is the angle between the edge along the z-axis (line ab) and the diagonal from the origin to the opposite corner (line ad)?

Figure 3



A) 54.7°
B) 45.0 °
C) 60.0 °
D) 22.5 °
E) 30.0 °

Ans:

$$\vec{A} = \overrightarrow{ab} = L\hat{k}$$

$$\vec{B} = \overrightarrow{ad} = L\hat{i} + L\hat{j} + L\hat{k}$$

$$\cos\phi = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{L^2}{L^2\sqrt{3}}$$

$$\cos\phi = \frac{1}{\sqrt{3}} = 54.7°$$

Q8.

If $\vec{A} = 1.0\hat{i} + 4.0\hat{j}$, $\vec{B} = -1.0\hat{j} + 2.0\hat{k}$ and $\vec{C} = 5.0\hat{i} - 1.0\hat{k}$. What is $2\vec{A}.[(\vec{B}\times\vec{A}) + \vec{C}]$?

A) 10
B) Zero
C) 28
D) 1.0
E) -32

Ans:

$\vec{B}\times\vec{A} = \hat{k} + 2\hat{j} - 8\hat{i}$

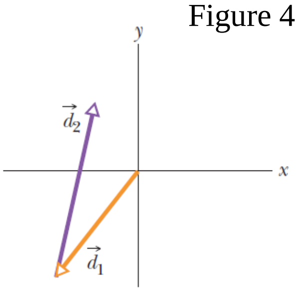
$(\vec{B}\times\vec{A}) + \vec{C} = -3\hat{i} + 2\hat{j}$

$2\vec{A}\cdot[(\vec{B}\times\vec{A}) + \vec{C}] = -6 + 16 = 10$

Q9.

The two vectors $\vec{d}_1$ and $\vec{d}_2$ lie in an x-y plane, as shown in **Figure 4**. What is the sign of the y component of $(\vec{d}_1 + \vec{d}_2)$, $(\vec{d}_1 - \vec{d}_2)$, and $(\vec{d}_2 - \vec{d}_1)$, respectively?

Figure 4



A) positive, negative, positive
B) negative, negative, positive
C) positive, positive, positive
D) positive, positive, negative
E) negative, positive, negative

Ans:

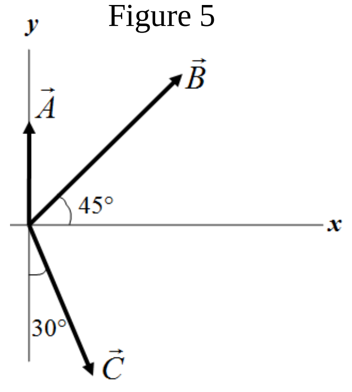
A

Q10.

Three displacement vectors ($\vec{A}, \vec{B}$ and $\vec{C}$) are shown in **Figure 5**, where the magnitude of the vectors are A = 20.0 cm, B = 40.0 cm and C = 30.0 cm. Find the resultant vector.

A) $(43.3\ \hat{i} + 22.3\ \hat{j})$ cm
B) $(54.3\ \hat{i} + 18.3\ \hat{j})$ cm
C) $(54.3\ \hat{i} + 28.3\ \hat{j})$ cm
D) $(28.3\ \hat{i} + 28.3\ \hat{j})$ cm
E) $(60.0\ \hat{i} + 20.0\ \hat{j})$ cm

Figure 5



Ans:

$\vec{A} = 0\,\hat{\imath} + 20\,\hat{\jmath}$

$\vec{B} = 4\cos 45\ \hat{\imath} + 40\sin 45\,\hat{\jmath}$

$\vec{C} = 30\sin 30\ \hat{\imath} - 30\cos 30\,\hat{\jmath}$

$\vec{R} = x\,\hat{\imath} + y\,\hat{\jmath}$

$x = 40\cos 45\ + 30\sin 30$

$y = 20\hat{\jmath} + 40\sin 45 - 30\cos 30$

$\vec{R} = 43.3\,\hat{\imath} + 22.3\,\hat{\jmath}$

Q11.

You are to launch a rocket, from just above the ground, with the following five initial velocities. Which one of them gives the rocket maximum horizontal range?

A) $(20.0\hat{i} + 20.0\ \hat{j})$ m/s
B) $(10.2\hat{i} + 26.4\ \hat{j})$ m/s
C) $(15.0\hat{i} + 24.0\ \hat{j})$ m/s
D) $(25.0\hat{i} + 13.3\ \hat{j})$ m/s
E) $(4.80\hat{i} + 27.9\ \hat{j})$ m/s

Ans:

Maximum horizontal range is obtained when $\theta = 45°$

Q12.

A particle leaves the origin with an initial velocity $\vec{v}_o = 2.0\hat{i}$ and a constant acceleration $\vec{a} = (-1.0\hat{i} + 2.0\,\hat{j})\,m/s^2$. By the time it reaches its maximum x coordinate, what is its average speed along y-direction?

A) 2.0 m/s
B) 3.6 m/s
C) 1.0 m/s
D) 1.6 m/s
E) 0.6 m/s

Ans:

$$v_x = v_{0x} + a_x t = 0$$

$$a_x = -1 \Rightarrow 2 - t = 0 \Rightarrow t = 2$$

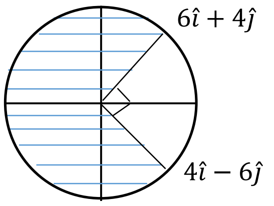
$$\Delta y = v_{0y} t + \frac{1}{2} a_y t2 = 0 + \frac{1}{2} \cdot 2 \cdot 4 = 4\,m$$

$$v_{avg} = \frac{\Delta y}{\Delta t} = \frac{4}{2} = 2.0\ m/s$$

Q13.

At t_1 = 2.0 s, the acceleration of a particle in counterclockwise circular motion is $(6.0\hat{i} + 4.0\,\hat{j})\,m/s^2$. It moves at constant speed. At time t_2 = 5.0 s, the particle's acceleration is $(4.0\hat{i} - 6.0\,\hat{j})\,m/s^2$. What is the radius of the path taken by the particle if t_2-t_1 is less than one period?

A) 2.9 m
B) 6.5 m
C) 7.2 m
D) 1.6 m
E) 0.2 m



Ans:

$$6.0\hat{i} + 4\,\hat{j} \Rightarrow 40\,\hat{i} - 60\hat{j}\ (3/4\ \text{Circle})$$

$$\Delta t = 5 - 2 = \frac{3}{4}\,T \Rightarrow T = 4s$$

$$v = \frac{2\pi r}{T}\ , a = \frac{v^2}{r} \Rightarrow r = \frac{4\pi^2 r^2}{aT^2} \Rightarrow r = \frac{aT^2}{4\pi^2} = \frac{(\sqrt{6^2 + 4^2}\,)4^2}{4\pi^2} = 2.9\,m$$

Q14.

After flying for 15 min in a wind blowing 44 km/h at an angle of 30° south of east, an airplane pilot is over a town that is 55 km due north of the starting point. What is the speed of the airplane relative to the wind?

A) 245 km/h
B) 38.1 km/h
C) 202 km/h
D) 220 km/h
E) 44.0 km/h

Ans:

$$\vec{V}_{PG} = \vec{V}_{PW} + \vec{V}_{WG}$$

$$\vec{V}_{PW} = \vec{V}_{PG} - \vec{V}_{WG}$$

$$\vec{V}_{PW} = 220\hat{j} - (44\cos 30\hat{i} - 44\sin 30\hat{j})$$

$$\vec{V}_{PW} = -38.1\hat{i} + 242\hat{j}$$

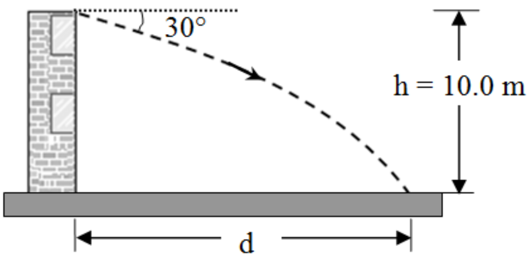
$$|\vec{V}_{PW}| = 245$$

Q15.

You throw a ball from a window at a height h =10.0 m above the ground, with an initial speed of 20 m/s at an angle 30° below the horizontal, see **Figure 6**. At what horizontal distance d will the ball hit the ground? Ignore air resistance.

A) 12.7 m
B) 34.6 m
C) 22.4 m
D) 10.0 m
E) 20.0 m

Figure 6



Ans:

$$y = V_0 \sin\theta_0 t - \frac{1}{2} g t^2$$

$$-10 = -20 \cdot \frac{1}{2}\, t - 4.9\, t^2$$

$$4.9\, t^2 - 10\, t - 10 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = 0.73 \text{ s}$$

$$x = v_{0x} t = 20 \cos 30 (0.73 \text{ s}) = 12.7 \text{ m}$$
