Q1.

The body mass index (BMI) of a person is calculated in SI units using the formula:

$$\text{BMI} = \text{weight (kg)} / \text{height}^2\ (\text{m}^2)$$

Find the BMI of a person (in SI units) whose weight is 160 lb (pound) and height is 70.0 inches. (1.00 inch = 2.54 cm, 1.00 lb = 454 g).

A) 23.0
B) 16.7
C) 5.45
D) 35.0
E) 45.2

Ans:

$$\text{BMI} = \frac{(160 \times 0.454)}{(70 \times 2.54 \times 10^{-2})^2} = 22.98$$

Q2.

It is observed that the frequency f (s^{-1}) of oscillations of a string depends upon its mass (M), length (L) and tension P (kg.m /s^2) as follows:

$$f = C\, P^a\, M^b\, L^c$$

where C is a dimensionless constant. Find the values of the constants a, b, and c (in this order)

A) ½, -½, -½
B) ½, ½, -½
C) -½, ½, ½
D) ½, -½, ½
E) -½, -½, ½

Ans:

$$[f] = [P]^a\,[m]^b\,[l]^c \Rightarrow T^{-1} = M^a\,L^a\,T^{-2a}M^{\,b}L^c$$

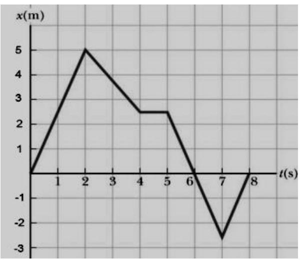
$$\Rightarrow -2a = -1\,, a + b = 0, a + c = 0$$

$$\Rightarrow a = -1/2, b = ½\,, c = 1/2$$

Q3.

The position versus time for a certain particle moving along the x-axis is shown in **Figure 1**. The average velocity in the time interval 4.0 s to 7.0 s is:

Figure 1



A) – 1.7 m/s
B) Zero
C) 1.7 m/s
D) 0.80 m/s
E) – 0.80 m/s

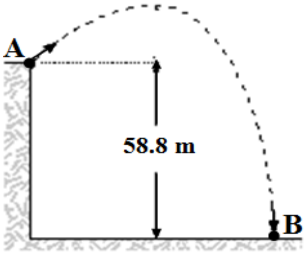
Ans:

$$v = (x_2 - x_1)/(t_2 - t_1) = (-2.5 - 2.5)/3 = -1.7 \text{ m/s}$$

Q4.

A stone is thrown outward from point A at the top of a 58.8 m high cliff with an upward velocity component of 19.6 m/s (see **Figure 2**). Assume that it lands on the ground, at point B, below the cliff, and that the ground below the cliff is flat. How long was the stone in the air? [Neglect the air resistance].

Figure 2



A) 6.00 s
B) 5.00 s
C) 4.00 s
D) 7.00 s
E) 8.00 s

Ans:

Use the equation

$$y = v_{yo}t - \frac{1}{2}gt^2$$

With $v_{yo} = 19.6$ m/s, $y = -58.8$ m, we have the equation:

$-58.8 = 39.2t - \frac{1}{2}9.8t^2$ which has the solution

solve $[9.8t^2 - 39.2\,t - 117.6 = 0, \quad \{t\}]$

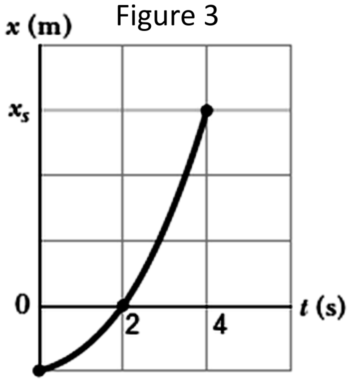
$\{\{t \rightarrow .-2.\}, \{t \rightarrow 6.\}\}$,

Q5.

Figure 3 illustrates the motion of a particle starting from rest and moving along an x-axis with a constant acceleration. The figure's vertical scaling is set by x_s = 12 m. The particle's acceleration is

A) 2.0 m/s^2
B) 0.50 m/s^2
C) – 6.0 m/s^2
D) 6.0 m/s^2
E) –3.0 m/s^2

Figure 3



Ans:

$x - x_0 = v_0 t + 1/2\,(a) t^2$

From the figure $x_0 = -4$

at t= 2
$0-(-4)=2v_0+2a$
$v_0+a= 2$ (1)

$at\ t = 4$
$12-(-4)) = 4v_0 + 8a$
$v_0 + 2a = 4$ (2)
$From\ (1)\ and\ (2) \qquad a = 2\ m/s^2$ #

Q6.

A ball is thrown directly downward from a height of 30.0 m. It takes 1.79 s to reach the ground. Find the magnitude of the initial velocity.

A) 7.99 m/s
B) 1.66 m/s
C) 10.0 m/s
D) 2.00 m/s
E) 3.75 m/s

Ans: $t = 1.79\ s$
Solve the equation for v0
$y-30 = v0\ t-1/2\,(9.80)t^2$
for v0, one finds
$v0 = 7.99\ m/s$ #

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Solve[9.8 × 1.79^2 / 2 - v0 1.79 - 30 == 0, {v0}]
{{v0 → -7.98878}}
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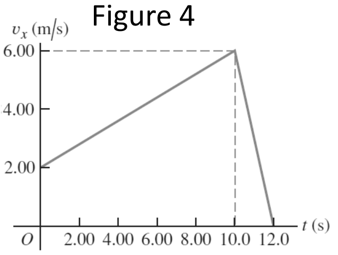
Q7.

A man is running in a straight line (along the x-axis). The graph in **Figure 4** shows the man's velocity as a function of time. During the first 12.0 s, the total distance traveled is

Figure 4



A) 46.0 m
B) Zero
C) 40.0 m
D) 8.00 m
E) 72.0 m

Ans:

$\text{Area} = 2 \times 10 + 1/2 \times 10 \times 4 + 1/2 \times 2 \times 6 = 46 \text{ m}$

Another solution

In the first 10 s

$a = 0.4 \text{ m/s}^2, x - x_0 = v_0 t + 1/2\, a_x t^2$

$x - x_0 = 2 \times 10 + 1/2\,(0.4)\,100 = 40\, m$

during the interval 10 to 12 s, the acceleration is $-6/2 = -3\, m/s^2$

$x - x_0 = 6 \times 2 - 1/2(3)4 = 6$

The total distance travelled is 40+6=46

Q8.

If $\vec{A} = \hat{i} - \hat{j}$ and $\vec{B} = 3.0\,\hat{i} + 2.0\,\hat{j}$, what is the angle between the two vectors? [$\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors in the x, y and z-direction, respectively]

A) 79°
B) 41°
C) 90°
D) 19°
E) 26°

Ans:

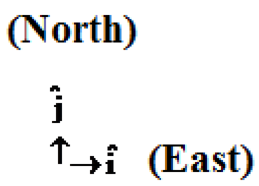
$\vec{A} \cdot \vec{B} = |A||B|\cos\psi$

$\therefore (1 - 3 - 1 \times 2) = \sqrt{2}\sqrt{13}\,\cos\psi \Rightarrow \psi = 79°$

Q9.

A boat is sailing due East at a speed of 6.0 m/s relative to the water of a river. The water is moving due south at a speed of 5.0 m/s relative to the ground. What is the velocity of the boat relative to the ground in unit vectors? See **Figure 5**. [$\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors in the x, y and z-direction, respectively]

Figure 5



A) $\vec{V}_{bg} = 6.0\,\hat{i} - 5.0\,\hat{j}$
B) $\vec{V}_{bg} = 3.0\,\hat{i} - 4.0\,\hat{j}$
C) $\vec{V}_{bg} = 8.0\,\hat{i} - 5.0\,\hat{j}$
D) $\vec{V}_{bg} = 6.0\,\hat{i} - 8.0\,\hat{j}$
E) $\vec{V}_{bg} = 5.0\,\hat{i} - 4.0\,\hat{j}$

Ans:

$V_{bw} = 6.0i$

$V_{wg} = -5.0j$

$V_{bg} = V_{bw} + V_{wg}$

$V_{bg} = 6.0i - 5.0j$

Q10.

A vector in the *xy* plane has a magnitude of 25 and the magnitude of its *x*-component is 12. The angle this vector makes with the positive *y*-axis is:

A) 29°
B) 64°
C) 61°
D) 24°
E) 41°

Ans:

$A_x = 12 = x\cos\psi = 25\cos\psi$

$\cos\psi = \frac{12}{25} \Rightarrow \psi = 61.3^\circ$

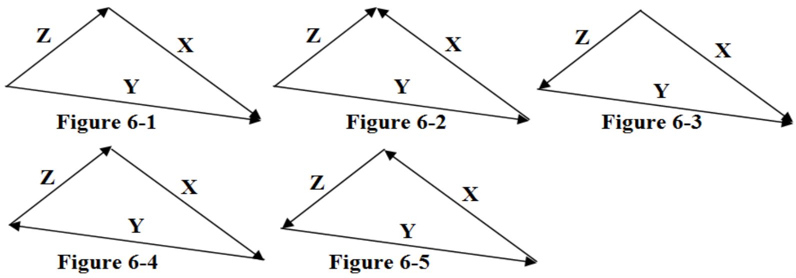
Then $90 - \psi \approx 29^\circ$

Q11.

The vectors **X**, **Y**, and **Z** are related by **Z - Y + X = 0**. Which diagram in **Figure 6** illustrates this relationship?

Figure 6

A) Figure 6-1
B) Figure 6-2
C) Figure 6-3
D) Figure 6-4
E) Figure 6-5



Ans:

Figure 6-1

Q12.

The result of $(\hat{j}\times\hat{k})\times(\hat{k}\times\hat{i})$ is:

[$\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors in the x, y and z-direction, respectively]

A) $\hat{k}$
B) 0
C) $\hat{i}$
D) $\hat{j}$
E) $-\hat{k}$

Ans:

$$(\hat{j}\times\hat{k})\times(\hat{k}\times\hat{i}) = \hat{i}\times\hat{j} = \hat{k}$$

Q13.

A particle undergoes a displacement, $\Delta\vec{r}=2.0\hat{i}-3.0\hat{j}+6.0\hat{k}$, ending with the position vector, $\vec{r}=3.0\hat{j}-4.0\hat{k}$ in meters. What was the particle's initial position vector? [$\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors in the x, y and z-direction, respectively]

A) $-2.0\hat{i}+6.0\hat{j}-10\hat{k}$
B) $6.0\hat{j}+10\hat{k}$
C) $2.0\hat{i}+3.0\hat{k}$
D) $2.0\hat{k}$
E) $-2.0\hat{i}+3.0\hat{j}-9.0\hat{k}$

Ans:

$$\Delta r = \hat{r}_f - \hat{r}_i = \Delta\vec{r} = 2.0\hat{i} - 3.0\hat{j} + 6.0\hat{k} = (3.0\hat{j} - 4.0\hat{k}) - \hat{r}_i$$

$$\hat{r}_i = 2.0\hat{i} + 6.0\hat{j} - 10\hat{k}$$

Q14.

A stone is tied to a string and rotated in a circle of radius 4 m at a constant speed. If the magnitude of its acceleration is 16 m/s^2, what is the period of the motion?

A) π s
B) 2π s
C) 3π s
D) π/2 s
E) 4π s

Ans:

$$V = \sqrt{R.a} = \sqrt{(4.0m \times 16\,m/s2)} = 8.0\,m/s$$
$$T = 2\pi R/v = (2\pi \times 4.0m)/(8.0\,m/s) = \pi\,s$$

Q15.

The minimum speed of a projectile during the whole flight is 5.0 m/s. It takes 4.0 s to reach its horizontal range. What is the horizontal range of the projectile?

A) 20 m
B) 30 m
C) 40 m
D) 50 m
E) 10 m

Ans:

The minimum speed of the projectile motion is at the highest point, when Vy =zero.
So minimum speed = Vx

Horizontal range=$V_x \times t$ = 5 m/s × 4 s =20 m
