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Q1.

A car travels with constant acceleration along a straight road. How much time does the car take to increase its speed from 30 m/s to 50 m/s in a distance of 180 m?

- A) 4.5 s
- B) 6.0 s
- C) 3.6 s
- D) 7.2 s
- E) 9.0 s

Ans:

$$v^2 = v_i^2 + 2ax$$

$$a = \frac{v^2 - v_i^2}{2x} = \frac{2500 - 900}{360} = 4.4 \text{ m/s}^2$$

$$v = v_i + at$$

$$t = \frac{v - v_i}{a} = \frac{50 - 30}{4.4} = 4.5 \text{ s}$$

Q2.

Consider three vectors \vec{A} , \vec{B} , \vec{C} such that $\vec{C} = \vec{A} + \vec{B}$. Which of the following operations will not change the magnitude of \vec{C} ?

- A) Rotate \vec{A} and \vec{B} each through the same angle about the same axis
- B) Multiply \vec{A} by 2 and divide \vec{B} by 2
- C) Divide \vec{A} by 2 and divide \vec{B} by 2
- D) Replace B by -B
- E) Replace \vec{A} by $-\vec{A}$

Ans:

$$C = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

A)
$$\checkmark$$
B) $C = \sqrt{4A^2 + \frac{B^2}{4} + 2AB \cos\theta} \times$

C) Same as (B)

$$D) \qquad \stackrel{\overrightarrow{B}}{\longleftrightarrow} \overrightarrow{A} \qquad \Rightarrow \qquad \stackrel{\overrightarrow{B}}{\longleftrightarrow} \qquad \overrightarrow{A}$$

E) same as (D)

Q3.

A ball is thrown horizontally from the top of a building that is 20.0 m high with a speed of 30.0 m/s. What is the speed of the ball when it hits the ground?

- A) 35.9 m/s
- B) 9.80 m/s
- C) 37.3 m/s
- D) 30.0 m/s
- E) 38.6 m/s

Ans:

$$Y := v_y^2 = 0 - 2g(y - y_0) = 2gh$$

$$v_y = \sqrt{2 \times 9.8 \times 20} = -19.8 \text{ m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = [(30)^2 + (19.8)^2]^{\frac{1}{2}} = 35.9 \text{ m/s}$$

Q4.

Two cars (A and B) are travelling due east on a highway. Their speeds relative to the ground are: $v_A = 30$ km/h and $v_B = 50$ km/h. What is the velocity of A relative to B?

- A) 20 km/h due west
- B) 20 km/h due east
- C) 80 km/h due west
- D) 80 km/h due east
- E) 40 km/h due east

$$\vec{v}_{AG} = \vec{v}_{AB} + \vec{v}_{BG}$$

$$\vec{v}_{AB} = \vec{v}_{AG} - \vec{v}_{BG}$$

$$= (30\hat{i}) - (50\hat{i}) = -20\hat{i} \Rightarrow 20 \text{ km/h due west}$$

Q5.

A force **F** pushes on three blocks on a frictionless surface, as shown in **Figure 1**. What is the magnitude of the force of block B on block A?



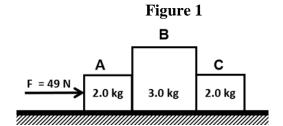
- C) 63 N
- D) 21 N
- E) 33 N

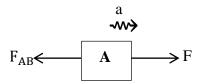
Ans:

$$a = \frac{F}{M} = \frac{49}{7} = 7.0 \text{ m/s}^2$$

$$m_A a = F - F_{AB}$$

$$\Rightarrow F_{AB} = F - m_A a = 49 - (2 \times 7) = 35 \text{ N}$$





Q6.

A box is sliding down an incline that is 35° above the horizontal. If the coefficient of kinetic friction between the block and the surface is 0.40, the magnitude of its acceleration is

A)
$$2.4 \text{ m/s}^2$$

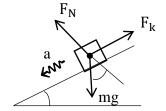
- B) 5.6 m/s^2
- C) 8.8 m/s^2
- D) 1.3 m/s^2
- E) zero

$$f_k = \mu_k F_N = \mu_k mg\cos\theta$$

$$ma = mgsin\theta - f_k$$

$$ma = mgsin\theta - \mu_k mg cos \theta$$

$$a = g(\sin\theta - \mu_k \cos\theta) = 2.4 \text{ m/s}^2$$



Q7.

A 2000-kg elevator starts from rest and accelerates upward at 3.20 m/s². How much power is delivered by the tension in the cable pulling the elevator at the instant when it has a speed of 6.00 m/s?

- A) 156 kW
- B) 221 kW
- C) 64.0 kW
- D) 193 kW
- E) 48.0 kW

Ans:

$$ma = T - mg$$

$$T = m(g + a) = (2000)(9.8 + 3.2) = 26 \times 10^3 \text{ N}$$

$$P_T = T \cdot v = 26 \times 10^3 \times 6.00 = 156 \text{ kW}$$

Q8.

A 3.0-kg block starts from rest and slides down a frictionless 30° incline, where it collides with a massless spring of force constant 400 N/m, as shown in Figure 2. The block slides a total distance of 0.65 m on the incline until it is stopped by the spring. By how much is the spring compressed?

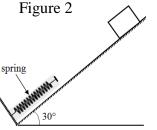
- A) 0.22 m
- B) 0.37 m
- C) 0.13 m
- D) 0.48 m
- E) 0.31 m

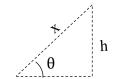
Ans:
$$\Delta K + \Delta U_g + \Delta U_s = 0$$

$$-mgh + \frac{1}{2}kd^2 = 0$$

$$\Rightarrow$$
 d = $\sqrt{\frac{2mgh}{k}} = \sqrt{\frac{mgx}{k}}$

$$= \sqrt{\frac{3 \times 9.8 \times 0.65}{400}} = 0.22 \text{ m}$$

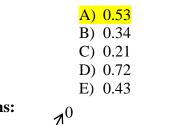




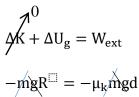
$$\sin\theta = \frac{h}{x} \Rightarrow h = x \cdot \sin\theta = \frac{x}{2}$$

Q9.

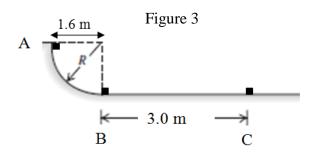
In **Figure 3**, a block is released from rest at point A and comes to rest at point C. The track from A to B is frictionless, while the track from B to C is rough. What is the coefficient of kinetic friction between the block and track BC?



Ans:

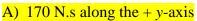


$$\mu_{\rm k} = \frac{R}{d} = \frac{1.6}{3} = 0.53$$

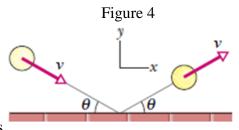


Q10.

A 4.0-kg object moving with speed 30 m/s strikes a surface at angle $\theta = 45^{\circ}$ and rebounds at the same angle with the same speed (see **Figure 4**). The impulse on the object is



- B) 170 N.s along the -y-axis
- C) 340 N.s along the + y-axis
- D) 340 N.s along the -y-axis
- E) 140 N.s at 45° relative to the + x-axis



$$\vec{\mathbf{p}}_i = mv \cos\theta \,\hat{\mathbf{i}} - mv \sin\theta \,\hat{\mathbf{j}}$$

$$\vec{\mathbf{p}}_{f} = mv \cos\theta \,\hat{\mathbf{i}} + mv \sin\theta \,\hat{\mathbf{j}}$$

$$\vec{\mathbf{j}} = \Delta \vec{\mathbf{p}} = \vec{\mathbf{p}}_{f} - \vec{\mathbf{p}}_{i} = 2 \text{ mv sin} \theta \,\hat{\mathbf{j}}$$

$$= 2 \times 4 \times 30 \times \frac{\sqrt{2}}{2}\hat{\mathbf{j}} = 170 \,\hat{\mathbf{j}} \,(\text{N} \cdot \text{s})$$

Q11.

A 4.0-kg object has a velocity of 4.0 m/s in the positive x direction when it explodes into two objects each with a mass of 2.0 kg. After the explosion, one of the objects has a velocity of 3.0 m/s at an angle of 60° measured counterclockwise from the positive x axis. What is the speed of the other object after the explosion?

- A) 7.0 m/s
- B) 8.9 m/s
- C) 7.9 m/s
- D) 6.1 m/s
- E) 6.7 m/s

Ans:

$$\vec{\mathbf{P}}_{i} = M\vec{\mathbf{V}}_{i} = (4 \times 4 \hat{\mathbf{i}}) = 16 \hat{\mathbf{i}}$$

$$\vec{\mathbf{p}}_1 = (2)(3\cos 60^\circ \hat{\mathbf{i}} + 3\sin 60^\circ \hat{\mathbf{j}}) = 3\hat{\mathbf{i}} + 5.2\hat{\mathbf{j}}$$

$$\vec{\mathbf{p}}_2 = \vec{\mathbf{P}}_i - \vec{\mathbf{p}}_1 = 13 \,\hat{\mathbf{i}} - 5.2 \hat{\mathbf{j}}$$

$$\vec{\mathbf{v}}_2 = \frac{\vec{\mathbf{p}}_2}{m_2} = 6.5 \,\hat{\mathbf{i}} - 2.6 \,\hat{\mathbf{j}}$$

$$v_2 = \sqrt{(6.5)^2 + (2.6)^2} = 7.0 \text{ m/s}$$

Q12.

A uniform solid sphere has a radius of 1.50 m. An applied torque of 9.50 N.m gives the sphere an angular acceleration of 6.00 rad/s² about a fixed axis through its center. What is the mass of the sphere?

- A) 1.76 kg
- B) 1.06 kg
- C) 1.41 kg
- D) 2.11 kg
- E) 1.59 kg

$$\tau = I\alpha \Rightarrow I = \frac{\tau}{\alpha}$$

$$I = \frac{2MR^2}{5} \Rightarrow \frac{2MR^2}{5} = \frac{\tau}{\alpha} \Rightarrow M = \frac{5\tau}{2 \times \alpha \times R^2} = \frac{5 \times 9.5}{2 \times 6 \times 2.25} = 1.76 \text{ kg}$$

Q13.

A particle is moving in a circle of radius 2.0 m with a tangential acceleration of 4.3 m/s². At an instant when the magnitude of the total acceleration is 6.0 m/s², what is the speed of the particle?

- A) 2.9 m/s
- B) 3.9 m/s
- C) 3.5 m/s
- D) 2.5 m/s
- E) 1.4 m/s

Ans:

$$a_{tot}^2 = a_t^2 + a_r^2$$

$$a_r^2 = a_{tot}^2 - a_t^2 = 36 - (4.3)^2 = 17.51$$

$$a_r = 4.18 \text{ m/s}^2$$

$$a_r = \frac{v^2}{R} \Rightarrow v = \sqrt{R \cdot a_r} = 2.9 \text{ m/s}$$

Q14.

Two equal particles, labeled **A** and **B** in **Figure 5**, are attached to a massless rod with a frictionless pivot at point **P**. The system is made to rotate clockwise with angular speed ω on a horizontal, frictionless tabletop. Particle **A** collides with and sticks to another equal particle that is at rest on the tabletop. What is the angular speed of the system immediately after the collision?

A) 0.56ω

- B) 0.60ω
- C) ω
- D) 0.82 ω
- E) 0.29ω

Ans:

$$I_i = MR^2 + M\left(\frac{R}{2}\right)^2 = \frac{5}{4}MR^2$$

$$I_f = (2M)R^2 + M\left(\frac{R}{2}\right)^2 = \frac{9}{4}MR^2$$

$$L_i = L_f \!\!: \ I_i \omega = I_f \omega_f \Rightarrow \omega_f = \frac{I_i}{I_f} \omega = \frac{5MR^2}{4} \cdot \frac{4}{9MR^2} \omega = 0.56 \ \omega$$

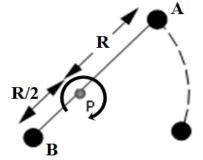


Figure 5

Q15.

A uniform beam has a weight of 120 N, and is supported as shown in **Figure 6**. What is the magnitude of the force by the pin on the beam?



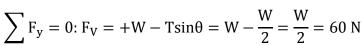
- B) 94 N
- C) 88 N
- D) 63 N
- E) 90 N

Ans:

$$\sum \tau_o = 0 \colon \, W \cdot \frac{L}{2} = T \cdot L \cdot sin\theta$$

$$T = \frac{W}{2\sin\theta} = \frac{120}{2 \times \sin 53} = 75.1 \text{ N}$$

$$\sum F_{x} = 0$$
: $F_{H} = T\cos\theta = 45.2 \text{ N}$



$$\Rightarrow$$
 F = $\sqrt{(45.2)^2 + (60)^2} = 75 \text{ N}$

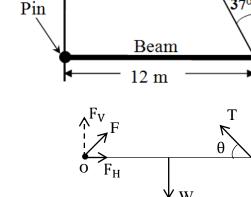


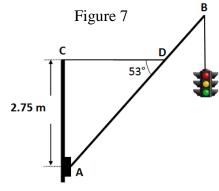
Figure 6

Q16.

A traffic light hangs from the structure shown in **Figure 7**. The uniform rod AB is 4.50 m long and has a mass of 5.00 kg. The mass of the traffic light is 10.0 kg. Determine the magnitude of the tension in the horizontal massless cable CD.



- B) 160 N
- C) 91.0 N
- D) 100 N
- E) 145 N



$$\sum \tau_A = 0:$$

$$5 \times 9.8 \times 2.25 \times \sin 37^{\circ} + 10 \times 9.8 \times 4.5 \times \sin 37^{\circ} = T \times 3.44 \times \sin 53^{\circ}$$

$$\Rightarrow$$
 T = 121 N

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Q17.

One end of a plastic rope, of length 45.0 m and radius 3.50 mm, is fixed to a ceiling while the other end is free. Its length increases by 1.10 m when a mass of 65.0 kg is attached to its free end. What is Young's modulus for plastic?

A)
$$6.78 \times 10^8 \text{ N/m}^2$$

B)
$$4.69 \times 10^8 \text{ N/m}^2$$

C)
$$6.25 \times 10^8 \text{ N/m}^2$$

D)
$$2.83 \times 10^8 \text{ N/m}^2$$

E)
$$8.54 \times 10^8 \text{ N/m}^2$$

Ans:

$$\frac{F}{A} = E \cdot \frac{\Delta L}{L}$$

$$E = \frac{F \cdot L}{A \cdot \Delta L} = \frac{mgL}{\pi r^2 \cdot \Delta L} = \frac{65 \times 9.8 \times 45}{\pi \times (3.5)^2 \times 10^{-6} \times 1.1} = 6.78 \times 10^8 \text{ N/m}^2$$

O18.

Two particles of mass M are initially separated by distance D. They are released from rest and accelerate toward each other through gravitational attraction. What is the kinetic energy of each particle when their separation is D/3?

A)
$$GM^2/D$$

B)
$$3 \text{ GM}^2/\text{D}$$

C)
$$GM/(2D^2)$$

E)
$$GM^2/2D$$

Ans:
$$U_i + K_i^0 = U_f + K_f$$

 $K_f = U_i - U_f = -\frac{GM^2}{D} + \frac{GM^2}{D/3} = 2\frac{GM^2}{D}$

$$\Rightarrow$$
 K(each) = $\frac{K_f}{2} = \frac{GM^2}{D}$

Q19.

An object is released from rest at a height h above the surface of a planet of mass M and radius R. What is the speed of the object just before striking the surface of the planet? Take h = 4000 km, R = 5000 km and $M = 4.0 \times 10^{24}$ kg.

- A) 6.9 km/s
- B) 7.8 km/s
- C) 3.5 km/s
- D) 5.4 km/s
- E) 4.8 km/s

Ans:

$$U_i + K_i^0 = U_f + K_f$$

$$K_f = U_i - U_f$$

$$\frac{1}{2}mv^2 = -\frac{GmM}{R+h} + \frac{GmM}{R}$$

$$v^2 = 2GM \left(\frac{1}{R} - \frac{1}{R+h}\right) = 2GM \left(\frac{1}{5 \times 10^6} - \frac{1}{9 \times 10^6}\right)$$

$$=2\times 6.67\times 10^{-11}\times 4\times 10^{24}\times 10^{-6}\left(\frac{1}{5}-\frac{1}{9}\right)$$

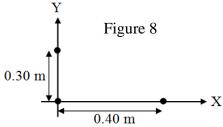
$$\Rightarrow$$
 v² = 4.74 × 10⁷ \Rightarrow v = 6.9 km/s

Q20.

Three particles, each with a mass of 5.0 kg, are located at points in the xy plane as shown in **Figure 8**. What is the magnitude of the gravitational force on the particle at the origin due to the other two particles?



- B) $2.7 \times 10^{-8} \text{ N}$
- C) $1.8 \times 10^{-8} \text{ N}$
- D) $3.4 \times 10^{-8} \text{ N}$
- E) $2.9 \times 10^{-8} \text{ N}$



$$F_{12} = \frac{6.67 \times 10^{-11} \times 25}{0.16} = 1.042 \times 10^{-8} \text{ N}$$

$$F_{13} = \frac{6.67 \times 10^{-11} \times 25}{0.09} = 1.853 \times 10^{-8} \text{ N}$$

$$F_{\text{net}} = (F_{12}^2 + F_{13}^2) = 2.13 \times 10^{-8} \text{ N}$$

Q21.

A planet moves around the Sun in the elliptical orbit shown in **Figure 9**. At point A, it is a distance of 1.75×10^8 km from the Sun and has a speed of 40 km/s. What is its speed at point B which is a distance of 2.50×10^8 km from the Sun?

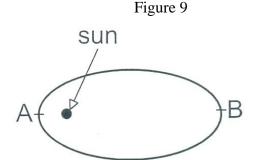


- B) 11 km/s
- C) 34 km/s
- D) 40 km/s
- E) 57 km/s

Ans:

$$l_A = l_B$$
: $m v_A r_A = m v_B r_B$

$$v_B = \frac{r_A}{r_B} v_A = \frac{1.75}{2.50} \times 40 = 28 \text{ km/s}$$



Q22.

The mass of a planet is 1/100 that of Earth and its radius is 1/4 that of Earth. If a person has a weight of 150 N on the surface of Earth, what would be his weight on the surface of the planet?

- A) 24 N
- B) 940 N
- C) 6.0 N
- D) 150 N
- E) 38 N

$$g = \frac{GM}{R^2}$$

$$g_p = \frac{GM_p}{R_p^2}$$

$$g_e = \frac{GMe}{Re^2}$$

$$g_R = \frac{GM_p}{R_p^2} \cdot \frac{R_e^2}{GMe} = \frac{M_p}{Me} \cdot \left(\frac{R_e}{R_p}\right)^2$$

$$=\frac{0.01Me}{Me} \cdot \left(\frac{R_e}{0.25R_e}\right)^2 = 0.16$$

$$W_p = m \cdot g_p = 0.16 \text{ mg}_e \implies W_p = 0.16 \times 150 = 24 \text{ N}$$

Q23.

Figure 10 shows a stream of water flowing through a hole at depth h = 10 cm in a tank holding water to height H = 50 cm and whose upper surface is open to the atmosphere. What is the speed of water exiting the hole?

- A) 1.4 m/s
- B) 2.3 m/s
- C) 3.5 m/s
- D) 1.8 m/s
- E) 2.9 m/s

Ans:

Bernoulli equation:

$$R_0 + \frac{1}{2} p v_t^2 + p g H = R_0 + \frac{1}{2} p v_b^2 + p g y$$

$$\Rightarrow V_b = \sqrt{2g(H - y)} = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.1} = 1.4 \text{ m/s}$$



The density of oil is 0.80 g/cm^3 . What is the height h of the column of oil shown in **Figure 11**?

- A) 10 cm
- B) 12 cm
- C) 2.0 cm
- D) 4.6 cm
- E) 8.0 cm

Ans:

$$\rho_w gx = \rho_0 gh$$

$$\rho_w(h-2) = \rho_0 h \Rightarrow \frac{h-2}{h} = \frac{\rho_0}{\rho_w} \Rightarrow 1 - \frac{2}{h} = 0.8$$

$$\Rightarrow \frac{2}{h} = 0.2 \Rightarrow h = \frac{2}{0.2} = 10 \text{ cm}$$

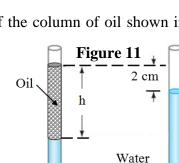


Figure 10

H

Q25.

An object has a weight of 30 N in air. It has a weight of 25 N when completely submerged in water. What is the volume of the object?

A)
$$5.1 \times 10^{-4} \text{ m}^3$$

- B) $4.6 \times 10^{-4} \text{ m}^3$
- C) $3.1 \times 10^{-4} \text{ m}^3$
- D) $2.6 \times 10^{-4} \text{ m}^3$
- E) $2.0 \times 10^{-4} \text{ m}^3$

Ans:

$$F_B = W_{air} - W_{water} = 5 N$$

$$\rho_{\text{W}} \cdot V_0 \cdot g = 5 \Rightarrow V_0 = \frac{5}{\rho_{\text{W}} g} = \frac{5}{9.8 \times 10^3} = 5.1 \times 10^{-4} \text{ m}^3$$

Q26.

Water, with a pressure of 3.5×10^5 Pa, is flowing at a speed of 5.0 m/s in a horizontal pipe. The area of the pipe is reduced to 1/3 its original value. What are the pressure and the speed of the water after the reduction?

- A) 2.5×10^5 Pa, 15 m/s
- B) 3.0×10^5 Pa, 10 m/s
- C) 3.0×10^5 Pa, 15 m/s
- D) 4.5×10^5 Pa, 1.5 m/s
- E) 5.5×10^5 Pa, 1.5 m/s

Ans:

Continuity equation: $A_i v_i = A_f v_f$

$$v_f = \frac{A_i}{A_f} \cdot v_i = \frac{3A_f}{A_f} \cdot (5) = 15 \text{ m/s}$$

Bernoulli equation: $p_i + \frac{1}{2}\rho v_i^2 = p_f + \frac{1}{2}\rho v_f^2$

$$\Rightarrow p_f = p_i + \frac{1}{2}\rho(v_i^2 - v_f^2) = 3.5 \times 10^5 + (500)(25 - 225) = 2.5 \times 10^5 P_a$$

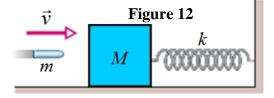
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Q27.

A block of mass M = 5.4 kg, at rest on a horizontal frictionless table, is attached to a rigid support by a spring of force constant k = 6000 N/m (see **Figure 12**). A bullet of mass m = 9.5 g and speed of 630 m/s strikes and is embedded in the block. What is the amplitude of the resulting harmonic motion?



- B) 2.8 cm
- C) 1.3 cm
- D) 7.6 cm
- E) 3.8 cm



Ans:

Conservation of momentum:

$$mv = (m + M)V$$

$$V = \frac{m}{m + M}v = \frac{9.5 \times 10^{-3}}{5.4095} \times 630 = 1.106 \text{ m/s}$$

$$\Rightarrow K_{\text{max}} = \frac{1}{2}(m+M)V^2 = 3.31 \text{ J}$$

$$K_{max} = U_{max}$$
: $K_{max} = \frac{1}{2}kx_m^2$

$$\Rightarrow$$
 x_m = $\sqrt{\frac{2K_{max}}{k}}$ = 3.3 cm

Q28.

A 0.50-kg mass connected to a spring is moving on a frictionless surface and its displacement is given by: $x(t) = 0.32 \cos(7.4t)$, where x is in meters and t is in seconds. What is the mechanical energy of the system?

- A) 1.4 J
- B) 0.90 J
- C) 0.47 J
- D) 4.4 J
- E) 0.19 J

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = m\omega^2 = 0.5 \times (7.4)^2 = 27.38 \frac{N}{m}$$

$$E = \frac{1}{2}kx_m^2 = \frac{1}{2} \times 27.38 \times (0.32)^2 = 1.4 \text{ J}$$

Q29.

A block attached to a spring oscillates in simple harmonic motion along the x-axis with amplitude x_m . Its total energy is 50.0 J. What is its kinetic energy when $x = x_m/2$?

- A) 37.5 J
- B) 12.5 J
- C) 25.0 J
- D) 50.0 J
- E) zero

Ans:

$$E = \frac{1}{2}kx_m^2 \Rightarrow k = \frac{2E}{x_m^2}$$

$$E = K + U$$

$$K = E - U = E - \frac{1}{2}kx^2 = \frac{1}{2}kx_m^2 - \frac{1}{2}kx^2$$

$$=\frac{k}{2}(x_{m}^{2}-x^{2})=\frac{k}{2}(x_{m}^{2}-0.25x_{m}^{2})$$

=
$$0.75 \left(\frac{1}{2} \text{kx}_{\text{m}}^2\right)$$
 = $0.75 \text{ E} = 0.75 \times 50 = 37.5 \text{ J}$

Q30.

A simple pendulum has a frequency of 3 Hz. To increase its frequency to 6 Hz

- A) Decrease its length by a factor of 4
- B) Decrease its length by a factor of 2
- C) Increase its length by a factor of 4
- D) Increase its length by a factor of 2
- E) Decrease its mass by a factor of 4

$$T = 2\pi \sqrt{\frac{l}{g}} \Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{g}{l}}$$