Q1.

The position function (x) of a particle moving along an x axis is $x = 1.0 + 8.0t - 2.0t^2$, with x in meters and the time (t) in seconds. Where does the particle stop momentarily?

A) 9.0 m
B) Zero
C) 1.0 m
D) 8.0 m
E) – 1.0 m

Ans:

$$x = 1 + 8t - 2t^2$$

$$v = 8 - 4t \longrightarrow 0 = 8 - 4t \longrightarrow t = 2\text{ s}$$

$$x = 1 + 16 - 8 = 9\text{ m}$$

Q2.

A particle has its position vector given by $\vec{r} = (2t)\hat{i} + (4t - 3t^2)\hat{j}$ (m), where t is the time in seconds. What is the magnitude of its average acceleration between t = 1 s and t = 3 s?

A) 6 m/s^2
B) Zero
C) 8 m/s^2
D) 2 m/s^2
E) 4 m/s^2

Ans:

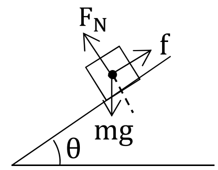
$$\vec{r} = 2t\hat{\imath} + (4t - 3t^2)\hat{\jmath}$$

$$\vec{v} = 2\hat{\imath} + (2 - 6t)\hat{\jmath}$$

$$\left.\begin{aligned}\vec{v}_1 &= 2\hat{\imath} - 4\hat{\jmath}\\ \vec{v}_3 &= 2\hat{\imath} - 16\hat{\jmath}\end{aligned}\right\}\vec{a}_{vg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{-12\hat{\jmath}}{2} = -6\hat{\jmath}\ m/s^2$$

Q3.

A block is initially at rest at the top of a rough inclined plane. The coefficients of friction between the block and the incline are $\mu_s = 0.500$ and $\mu_k = 0.400$. The angle θ between the incline and the ground is gradually increased. At what value of θ will the block start sliding down the incline?



A) 26.6°
B) 21.8°
C) 63.4°
D) 68.2°
E) 45.0°

Ans:

$y - \text{comp:} \quad F_N = \text{mg}\cos\theta$

$x - \text{comp:} \quad \text{mg}\sin\theta = f_s$

on the verge of sliding: $f_s = f_{sgmax} = \mu_s F_N$

$\Rightarrow \text{mg}\sin\theta = \mu_s \text{mg}\cos\theta \Rightarrow \tan\theta = \mu_s \Rightarrow \theta = \tan^{-1}(\mu_s)$

Q4.

A car moves on a horizontal circular road of radius 200 m. The coefficient of static friction between its tires and the road is 0.30. The car starts from rest and reaches the maximum speed with which it can travel without sliding in 5.0 seconds. The magnitude of the **total** acceleration is:

A) 5.7 m/s²
B) 4.8 m/s²
C) 9.0 m/s²
D) 7.8 m/s²
E) 2.9 m/s²

Ans:

$$\frac{mv^2}{R} = f_s = m\mu_g$$

$$v = \sqrt{\mu_s Rg}$$

$$v = v_i^{\,0} + at$$

$$a_t = \frac{v}{t} = \frac{\sqrt{\mu_s Rg}}{t} = \frac{\sqrt{0.3 \times 200 \times 9.8}}{5} = 4.849 m/s^2$$

$$a_r = \frac{v^2}{R} = \mu_s g = 0.3 \times 9.8 = 2.94\ m/s^2$$

$$a = \sqrt{a_t^2 + a_r^2} = 5.7 \text{ m/s}^2$$

Q5.

A truck of mass 2500 kg accelerates from rest to 12 m/s in a time t = 6.0 s with constant acceleration. Calculate the instantaneous power delivered by the engine at t = 5.0 s.

A) 41.7 kW
B) 15 kW
C) 25 kW
D) 5.5 kW
E) 10 kW

Ans:

$$a = \frac{\Delta v}{\Delta t} = \frac{10}{6} m/s^2 = \frac{5}{3} m/s^2$$

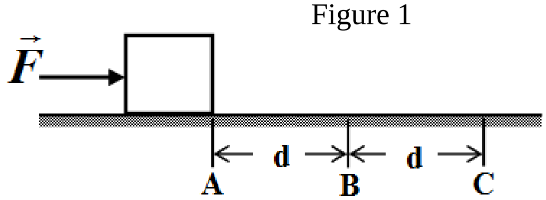
$$F = ma = \frac{12.5}{3} \times 10^3 \text{ N}$$

$$P = F \cdot v = \frac{12.5 \times 10^3}{3} \times 10 = 41.7 \text{ kW}$$

Q6.

A constant horizontal force (**F**) is exerted on a block that is free to slide on a frictionless surface as shown in **Figure 1**. The block starts from rest at point A, and when it reaches point B, it has speed v_B. When the block has travelled another distance d to point C, its speed is:

Figure 1



A) $\sqrt{2}\, v_B$
B) v_B
C) $v_B/\sqrt{2}$
D) $v_B/2$
E) $2v_B$

Ans:

$A \rightarrow B: \quad W = \Delta K$

$$W = \frac{1}{2} m v_B^2$$

$B \rightarrow C: \quad W = \frac{1}{2} m v_B^2$

$$\Delta K = \frac{1}{2} m v_C^2 - \frac{1}{2} m v_B^2$$

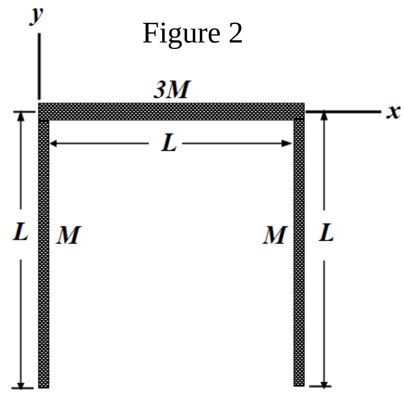
$W = \Delta K$:

$$\frac{1}{2} m v_B^2 = \frac{1}{2} m v_C^2 - \frac{1}{2} m v_B^2 \Rightarrow 2v_B^2 = v_C^2 \Longrightarrow v_C = \sqrt{2} v_B$$

Q7.

An object consists of three uniform thin rods, each of length L= 30 cm, that form an inverted U, as shown in **Figure 2**. The vertical rods each has a mass of M; the horizontal rod has a mass of 3M. What is the position vector of the center of mass of the object (in cm)?

Figure 2



A) $\vec{\mathbf{r}}_{com} = 15\,\hat{\mathbf{i}} - 6.0\,\hat{\mathbf{j}}$

B) $\vec{\mathbf{r}}_{com} = 15\,\hat{\mathbf{i}}$

C) $\vec{\mathbf{r}}_{com} = 15\hat{\mathbf{j}}$

D) $\vec{\mathbf{r}}_{com} = 15\,\hat{\mathbf{i}} - 15\,\hat{\mathbf{j}}$

E) $\vec{\mathbf{r}}_{com} = 15\,\hat{\mathbf{i}} - 7.5\,\hat{\mathbf{j}}$

Ans:

$$\left.\begin{array}{l} 1:\quad m_1 = M;\ x_1 = 0;\quad y_1 = -15 \\ 2:\quad m_2 = M;\ x_2 = 30;\quad y_2 = -15 \\ 3:\quad m_3 = 3M;\ x_3 = 15;\quad y_3 = 0 \end{array}\right\} M_{total} = 5\text{ M}$$

$$x_{com} = \frac{1}{5\cancel{M}}[0 + 30\,\cancel{M} + 45\,\cancel{M}] = 15\text{ cm}$$

$$y_{com} = \frac{1}{5\cancel{M}}[-15\,\cancel{M} - 15\,\cancel{M}] = -6\text{ cm}$$

Q8.

A disk of mass 1.0 kg and radius 2.0 m is rotating about an axis passing through its center and perpendicular to its plane. It is slowing down at the rate of 7.0 rad/s^2. The magnitude of the net torque acting on it is:

A) 14 N·m
B) 7.0 N·m
C) 28 N·m
D) 44 N·m
E) 23 N.m

Ans:

$$I_{com} = \frac{1}{2}MR^2 = \frac{1}{2} \times 1.0 \times 4.0 = 2.0\text{ kg.m}^2$$

$$\tau_{net} = I_{com} \cdot \alpha = 2 \times 7 = 14\ N \cdot m$$

Q9.

A uniform solid cylinder starts from rest at the top of a rough incline and rolls smoothly without sliding down the incline. What is the speed of its center of mass when it has descended by a vertical distance of 4.0 m?

A) 7.2 m/s
B) 8.5 m/s
C) 6.0 m/s
D) 5.5 m/s
E) 4.7 m/s

Ans:

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I_{com}\omega^2$$

$$= \frac{1}{2}mv^2 + \left(\frac{1}{2} \times \frac{1}{2}M\cancel{R}^2 \times \frac{V^2}{\cancel{R}^2}\right) = 0.75\, mv^2$$

$$\cancelto{0}{K_i} + U_i = K_f + \cancelto{0}{U_f}$$

$$0.75\cancel{m}v^2 = \cancel{m}gh$$

$$v^2 = \frac{gh}{0.75} \Rightarrow v = \sqrt{\frac{9.8 \times 4}{0.75}} = 7.2\, m/s$$

Q10.

A horizontal massless rod is used to support a weight W, as shown in **Figure 3**. The rod is supported by a cable at an angle θ from the horizontal and by a hinge at point *P*. The value of the horizontal component of the force exerted by the hinge is:

A) W.(cot θ)
B) W.(csc θ)
C) W.(tan θ)
D) W.(sin θ)
E) W.(cos θ)

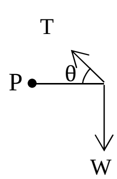
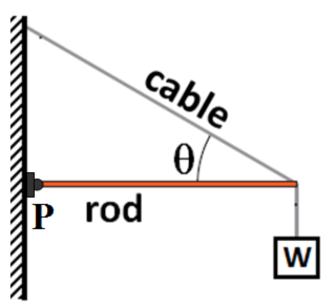


Figure 3



Ans:

$$\sum \tau_p = 0 \ : T \cdot L \cdot \sin\theta = W \cdot L$$

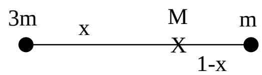
$$T = \frac{W}{\sin\theta}$$

$$\sum F_x = 0: \quad F_x = T_x = \frac{W}{\sin\theta} \cdot \cos\theta = W \cdot \cot\theta$$

Q11.

A particle of mass 3*m* is located 1.0 m from a particle of mass *m*. At what distance from *3m* should you place a third particle of mass *M* so that the net gravitational force on *M* due to the other two particles is zero?

A) 0.63 m
B) 2.4 m
C) 0.37 m
D) 0.42 m
E) 1.6 m



Ans:

$$\frac{G \cdot 3m \cdot M}{x^2} = \frac{G \cdot m \cdot M}{(1-x)^2}$$

$$\left(\frac{1-x}{x}\right)^2 = \frac{1}{3}$$

$$\frac{1}{x} - 1 = \sqrt{\frac{1}{3}} = 0.577$$

$$\frac{1}{x} = 1.577 \Rightarrow x = 0.63\ m$$

Q12.

The magnitude of the gravitational acceleration at the north pole of planet Neptune is 10.7 m/s^2. Neptune has a radius of 2.50×10^4 km and rotates once around its axis in 16.0 hours. What is the magnitude of the free fall acceleration at the equator of Neptune?

A) 10.4 m/s^2
B) 10.7 m/s^2
C) 11.0 m/s^2
D) 10.2 m/s^2
E) zero

Ans:

$$a_r = \frac{v^2}{R} = \frac{\omega^2 R^2}{R} = R\omega^2 = R \cdot \left(\frac{2\pi}{T}\right)^2 = \frac{4\pi^2 R}{T^2}$$

$$= 4\pi^2 \times 2.5 \times 10^7 \times (16 \times 3600)^{-2} = 0.297\ m/s^2$$

$$a_{eq} = a_{Np} - a_r = 10.7 - 0.297 = 10.4\ m/s^2$$

Q13.

A 1000-kg satellite moves in a circular orbit around the Earth at an altitude of 100 km. How much energy must be added to the satellite-Earth system to move the satellite into a circular orbit with altitude 200 km?

A) 4.69×10^{8} J
B) 6.38×10^{8} J
C) 1.99×10^{12} J
D) 9.97×10^{11} J
E) 3.08×10^{10} J

Ans:

$$E = -\frac{GmM}{2r}$$

$$\Delta E = -\frac{GmM}{2r_f} + \frac{GmM}{2r_i} = \frac{GmM}{2}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

$$= \frac{GmM}{2}\left(\frac{1}{h_i + R} - \frac{1}{h_f + R}\right)$$

$$= \frac{1}{2} \times 6.67 \times 10^{-11} \times 1000 \times 5.98 \times 10^{24} \times \left(\frac{1}{6.37 \times 10^6 + 10^5} - \frac{1}{6.37 \times 10^6 + 2 \times 10^5}\right)$$

$$= 4.69 \times 10^8 \text{ J}$$

Q14.

A spherical planet has a radius of 500 km. The acceleration due to gravity at the surface of the planet is 3.00 m/s^2. With what speed will an object hit the surface of the planet if it is dropped from rest from 300 km above the surface?

A) 1.06 km/s
B) 1.34 km/s
C) 2.19 km/s
D) 1.73 km/s
E) 1.58 km/s

Ans:

On the surface: $= a_g = \frac{GM}{R^2} \to GM = R^2\, a_g$

$$U_i + \cancelto{0}{K_i} = U_f + K_f$$

$$K_f = U_i - U_f : \frac{1}{2}\cancel{m}v^2 = -\frac{G\cancel{m}M}{R+h} + \frac{G\cancel{m}M}{R}$$

$$v^2 = \frac{2GM}{R} - \frac{2GM}{R+h} = 2GM\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$= 2\,R^2 \cdot a_g\left(\frac{1}{R} - \frac{1}{R+h}\right) \Rightarrow v = 1.06\ km/s$$

Q15.

A planet makes a circular orbit with period T around a star. If the planet were to orbit, at the same distance, around a star with three times the mass of the original star, what would be the new period?

A) $T/\sqrt{3}$
B) T
C) $3T$
D) $\sqrt{3}T$
E) $T/3$

Ans:

$$F_{centripetal} = F_{gravity}: \frac{mv^2}{R} = \frac{GmM}{R^2} \longrightarrow v^2 = \frac{GM}{R}$$
$$\longrightarrow \frac{1}{v^2} = \frac{R}{GM}$$

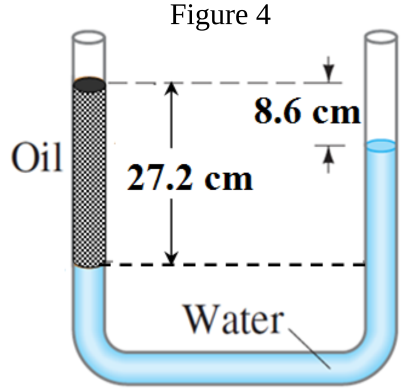
$$T = \frac{2\pi R}{v} \rightarrow T^2 = \frac{4\pi^2 R^2}{v^2} = 4\pi^2 R^2 \cdot \frac{R}{GM} = \frac{4\pi^2}{GM} \cdot R^3$$

$$\left.\begin{aligned} T^2 &= \frac{4\pi^2 R^3}{G} \cdot \frac{1}{M} \\ t^2 &= \frac{4\pi^2 R^3}{G} \cdot \frac{1}{3M} \end{aligned}\right\} \frac{t^2}{T^2} = \frac{1}{3M} \div \frac{1}{M} = \frac{M}{3M} = \frac{1}{3} \Rightarrow t = \frac{T}{\sqrt{3}}$$

Q16.

If the density of water is 1.00 g/cm^3, determine the density of the oil filled in the left column of the U-tube shown in **Figure 4**.

Figure 4



A) 684 kg/m^3
B) 581 kg/m^3
C) 755 kg/m^3
D) 322 kg/m^3
E) 475 kg/m^3

Ans:

$$p_A = p_o + \rho_x \cdot g \cdot (27.2)$$

$$p_B = p_o + \rho_w \cdot g \cdot (18.58)$$

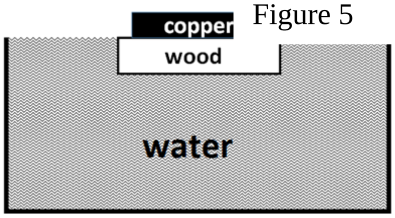
$$p_A = p_B : 27.2\, \rho_x = 18.58\, \rho_w$$

$$\Rightarrow \rho_x = \frac{18.58}{27.2}\, \rho_w = 0.684\ g/cm^3$$

Q17.

A piece of copper is placed on top of a 0.40-kg block of wood that is floating in water, as shown in **Figure 5**. The density of wood is 0.60×10^3 kg/m^3, and the system is in equilibrium. What is the mass of the copper piece if the top face of the wood block is exactly at the water's surface?

Figure 5



A) 0.27 kg
B) 0.40 kg
C) 0.13 kg
D) 0.55 kg
E) 0.90 kg

Ans:

H: water , w: wood , c: copper

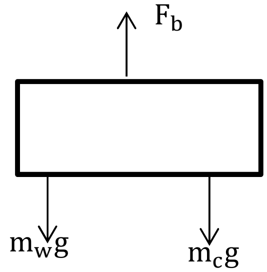
$F_b = m_w g + m_c g$

$\rho_H V_w g = m_w g + m_c g$

$m_c = \rho_H V_W - m_W$

$= \rho_H \cdot \frac{m_W}{\rho_W} - m_W = \left(\frac{\rho_H}{\rho_W} - 1\right) m_W$

$= \left(\frac{1000}{600} - 1\right) \times 0.4 = 0.266$ kg



Q18.

In a house, the water is pumped at a speed of 0.50 m/s through a pipe in the ground floor under a pressure of 3.0 atm. What is the pressure in a pipe on the top floor 5.0 m above where the water speed is 1.2 m/s?

A) 2.5×10^5 N/m^2
B) 3.5×10^5 N/m^2
C) 1.5×10^5 N/m^2
D) 1.7×10^5 N/m^2
E) 2.1×10^5 N/m^2

Ans:

Berroulli Equation:

g: ground floor , u: top floor

$p_g + \frac{1}{2}\rho v_g^2 + 0 = p_u + \frac{1}{2}\rho v_u^2 + \rho g y_u$

$p_u = p_g + \frac{1}{2}\rho(v_g^2 - v_u^2) - \rho g y_u$

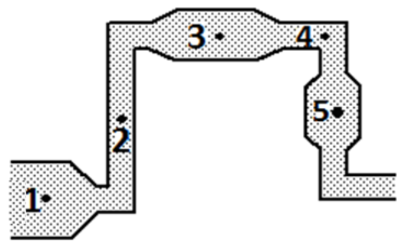
$= (3.0 \times 1.01 \times 10^5) + (500)(0.25 - 1.18^2) - (1000 \times 9.8 \times 5)$

$= 2.53 \times 10^5 P_a$

Q19.

A **horizontal** piping system that delivers a constant flow of water is constructed from pipes with different diameters as shown by the **top view** in **Figure 6**. At which of the labeled points is the water in the pipe under the greatest pressure?

Figure 6



A) **1**
B) **2**
C) **3**
D) **4**
E) **5**

Ans:

All point are at the same elevation.

$$\text{pressure} \propto \frac{1}{\text{speed}}$$

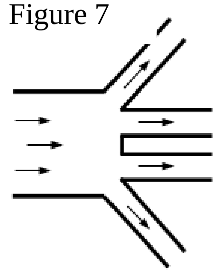
$$\text{speed} \propto \frac{1}{\text{area}}$$

$$\text{pressure} \propto \text{area}$$

Q20.

Water is flowing through a river that is 12 m wide with a speed of 0.75 m/s. The water then flows into four identical smaller rivers each having a width of 4.0 m, as shown in **Figure 7**. The depth of the water does not change as it flows into the four rivers. What is the speed of the water in one of the smaller rivers? Assume that the cross sectional areas of all rivers to be rectangular in shape.

Figure 7



A) 0.56 m/s
B) 0.75 m/s
C) 2.3 m/s
D) 0.25 m/s
E) 0.12 m/s

Ans:

d = depth of channel

Big channel: $\text{flow rate} = R_V = A.v = 12 \times d \times 0.75 = 9\text{ d}$

$$\text{Small channel: flow rate} = r_V = \frac{R_V}{4} = 2.25d$$

$$r_V = a \cdot v: \quad 2.25\not{d} = 4\not{d}v$$

$$\Rightarrow v = \frac{2.25}{4} = 0.56\ m/s$$

Q21.

A physical pendulum consists of a meter stick that is pivoted at a small hole drilled through a distance of 10 cm above the center of the stick. The period of oscillation is:

A) 1.9 s
B) 0.60 s
C) 0.32 s
D) 3.7 s
E) 1.6 s

Ans:

$$I = \frac{1}{12}mL^2 + mh^2 = m\left(\frac{1}{12} + 0.01\right) = 0.093\ m$$

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{0.093\ \not{m}}{\not{m} \times 9.8 \times 0.1}} = 1.9\ s$$

Q22.

An object is moving in simple harmonic motion on a horizontal frictionless surface. When the object is displaced 0.55 m to the right of its equilibrium position, it has an acceleration of 8.5 m/s^2 to the left. What is the angular frequency of the motion?

A) 3.9 rad/s
B) 2.3 rad/s
C) 3.1 rad/s
D) 2.7 rad/s
E) 5.3 rad/s

Ans:

$$a = -\omega^2 x \Rightarrow \omega = \sqrt{-\frac{a}{x}}$$

$$= \sqrt{-\frac{-8.5}{0.55}} = 3.9\ rad/s$$

Q23.

An object is undergoing simple harmonic motion with period 0.25 s and amplitude 5.0 cm on a horizontal frictionless surface. At time t = 0, the object is momentarily at rest at x = 5.0 cm. Find the shortest time it takes the object to go from x = 5.0 cm to x = –1.5 cm.

A) 0.075 s
B) 1.3 s
C) 2.9 s
D) 0.050 s
E) 0.025 s

Ans:

$$\mathrm{x} = x_m \cdot cos(\omega t + \Phi)$$

$$\mathrm{v} = -\omega x_m \cdot \sin(\omega t + \Phi)$$

$$0 = -\omega x_m \cdot sin\Phi \Rightarrow \Phi = 0$$

$$\Rightarrow \mathrm{x} = x_m \cdot \cos \omega t$$

$$x_0 = x_m = 5.0\ cm$$

$$-1.5 = 5.0 \times cos\omega t$$

$$-0.3 = cos\omega t$$

$$\omega t = \cos^{-1}(-0.3)$$

$$T = \frac{2\pi}{\omega} \rightarrow \frac{1}{\omega} = \frac{T}{2\pi}$$

$$t = \frac{1}{\omega} \cdot cos^{-1}(-0.3) = \frac{T}{2\pi} \cdot cos^{-1}(-0.3) = 0.075\ s$$

Q24.

A block-spring system, moving with simple harmonic motion on a horizontal frictionless surface, has an amplitude x_m. When the kinetic energy of the block equals twice the potential energy stored in the spring, what is the position x of the block?

A) $x_m/\sqrt{3}$
B) $x_m/3$
C) $x_m/2$
D) $3x_m$
E) $\sqrt{3}\,x_m$

Ans:

$$\mathrm{K} + \mathrm{U} = \mathrm{E}$$

$$2\mathrm{U} + \mathrm{U} = \mathrm{E} \quad \Rightarrow \quad 3\mathrm{U} = \mathrm{E}$$

$$3 \times \frac{1}{2}\mathrm{k} \times x^2 = \frac{1}{2}\mathrm{k} x_m^2$$

$$x^2 = \frac{x_m^2}{3}$$

Q25.

A 2.0 kg block connected to a spring executes simple harmonic motion on a horizontal frictionless surface. The position of the block is given by x = (1.0 cm) cos (9.4 t + π/2). The total mechanical energy of the system is:

A) 8.8 mJ
B) 1.8 mJ
C) 2.2 mJ
D) 1.1 mJ
E) 3.5 mJ

Ans:

$\omega = 9.4 \text{ rad/s}$

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = m\omega^2 = 2.0 \times (9.4)^2 = 176.72 \quad N/m$$

$$\text{E} = \frac{1}{2}kx_m^2 = \frac{1}{2} \times 176.72 \times 1.0 \times 10^{-4} = 8.8 \quad m\text{J}$$