Q1.

Gold, which has a density of 19.32 g/cm^3, can be pressed into a thin sheet or drawn out into a long wire. If a sample of gold with a mass of 26.57 g is pressed into a sheet of 1.100 μm thickness, what is the area of the sheet?

A) 1.250 m^2
B) 1.430 m^2
C) 1.132 m^2
D) 1.030 m^2
E) 1.530 m^2

Ans:

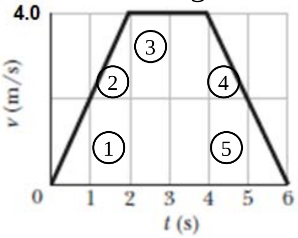
$$Volume\ V = \frac{mass\ m}{density\ \rho} = Area\ A\ \times thickness\ t\ \Rightarrow A = \frac{m}{\rho t}$$

$$A = \frac{26.57 \times 10^{-3}}{19.32 \times 10^{3} \times 1.1 \times 10^{-6}} = 1.250\ m^2$$

Q2.

A particle starts from the origin at t = 0 and moves along the positive x axis. A graph of the velocity of the particle as a function of time is shown in **Figure 1**. What is the average velocity of the particle between ***t* = 1.0 s** and ***t* = 5.0 s**?

Figure 1



A) + 3.5 m/s
B) – 1.1 m/s
C) + 2.7 m/s
D) – 7.2 m/s
E) + 8.7 m/s

Ans:

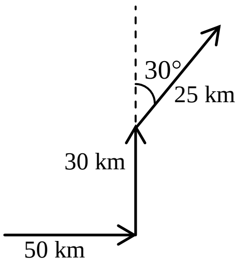
$$V_{avg} = \frac{\Delta X}{\Delta t} = \frac{Area\ ① + Area\ ② + Area\ ③ + Area\ ④ + Area\ ⑤}{5 - 1}$$

$$= \frac{2 + \frac{1 \times \cancel{2}}{\cancel{2}} + 8 + \frac{1 \times \cancel{2}}{\cancel{2}} + 2}{4} = \frac{14}{4} = 3.5\ \text{m/s}$$

Q3.

A car is driven east for a distance of 50 km, then north for 30 km, and then in a direction 30° east of north for 25 km. Calculate the magnitude of the car's total displacement from its starting point.

A) 81 km
B) 55 km
C) 97 km
D) 35 km
E) 11 km



Ans:

$$Total\ displacement\ \vec{D} = 50\vec{\imath} + 30\vec{\jmath} + 25\cos 30\vec{\jmath} + 25\sin 30\vec{\imath}$$

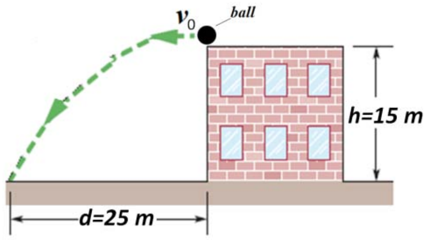
$$\vec{D} = (50 + 25\sin 30)\vec{\imath} + (30 + 25\cos 30)\vec{\jmath}$$

$$\vec{D} = 62.5\,\vec{\imath} + 51.65\,\vec{\jmath} \implies |D| = \sqrt{(62.5)^2 + (51.65)^2} = 81.08\,m \approx 81\,km$$

Q4.

A ball is thrown horizontally leftward from the left edge of a roof, at height h =15.0 m above the ground. The ball hits the ground at a distance d = 25.0 m from the building, as shown in **Figure 2**. Find the speed of the ball when it hits the ground. (Ignore air resistance).

Figure 2



A) 22.3 m/s
B) 17.2 m/s
C) 14.3 m/s
D) 32.8 m/s
E) 42.9 m/s

Ans:

$$|v_{fy}| = \sqrt{v_{1y}^2 - 2gy} = \sqrt{-2gh} = \sqrt{2 \times 9.8 \times 15} = 17.14\,m/s;\ v_{fy} = -17.14\,m/s$$

$$t = \frac{v_{fy} - v_{iy}}{-g} = \frac{v_{fy}}{-g} = \frac{\cancel{-}17.14}{\cancel{-}9.8} = 1.75\,s$$

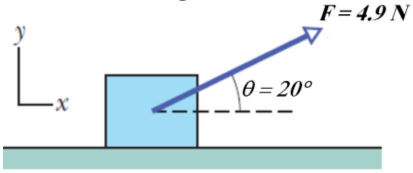
$$v_{fx} = \frac{d}{t} = \frac{25}{1.75} = 14.3\,m/s$$

$$|v_f| = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{(14.3)^2 + (-17.14)^2} = 22.3\,m/s$$

Q5.

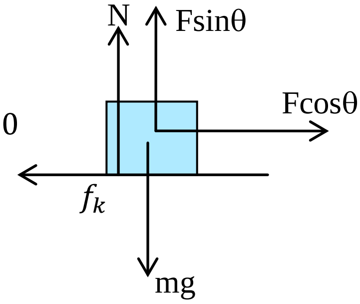
Figure 3 shows an initially stationary block of 1.00 kg mass on a rough floor. A force $\vec{F}$, of magnitude 4.90 N and making an angle θ = 20.0° with the horizontal, is then applied to the block. What is the magnitude of the acceleration of the block across the floor if the coefficient of kinetic friction μ_k = 0.300?

Figure 3



A) 2.17 m/s^2
B) 3.62 m/s^2
C) 5.73 m/s^2
D) 1.55 m/s^2
E) 1.01 m/s^2

Ans:



$$\sum F_x = Fcos\theta - f_k - ma = 0$$

$$a = \frac{Fcos\theta - \mu_k N}{m} \;;\; but \sum F_y = N + Fsin\theta - mg = 0$$

$$N = mg - Fsin\theta$$

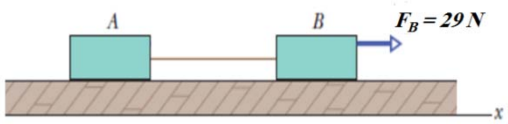
$$a = \frac{Fcos\theta - \mu_k(mg - Fsin\theta)}{m}$$

$$a = \frac{4.9 \times \cos 20 - 0.3(1 \times 9.8 - 4.9 \times sin20)}{1} = 2.17\ m/s^2$$

Q6.

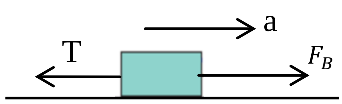
In **Figure 4**, block *A* with 4.0 kg mass and block *B* with 6.0 kg mass, lying on a frictionless horizontal surface, are connected by a string of negligible mass. When a single force of magnitude $F_B = 29N$ acts on block *B*, then both blocks move with a constant acceleration. What is the tension in the string?

Figure 4



A) 12 N
B) 16 N
C) 33 N
D) 9.1 N
E) 3.9 N

Ans:



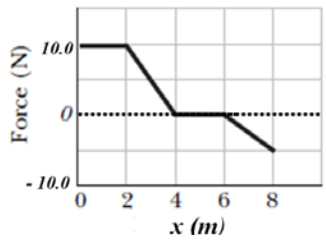
$$a = \frac{F_B}{m_A + m_B} = \frac{29}{4+6} = 2.9\ m/s^2$$

$$F_B - T = m_B a \Rightarrow T = F_B - m_B a = 29 - 6 \times 2.9 = 11.6 \approx 12\ \text{N}$$

Q7.

A 5.00 kg block moves in a straight line on a horizontal frictionless surface under the influence of a single force that varies with position *x* as shown in **Figure 5**. If the block has a speed *v* = 3.00 m/s at *x* = 0.00 m, find the speed of the block at *x* = 8.00 m.

Figure 5



A) 4.36 m/s
B) 2.16 m/s
C) 3.62 m/s
D) 7.73 m/s
E) 9.55 m/s

Ans:

$$\Delta K = W = Area;\ [Area]_{0-8m} = 2 \times 10 + \frac{\cancel{2} \times 10}{\cancel{2}} + 0 - \frac{\cancel{2} \times 5}{\cancel{2}} = 20 + 10 - 5 = 25\ J$$

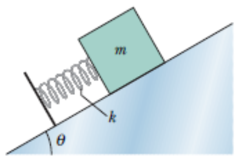
$$\Delta K = \frac{1}{2} m \left(v_f^2 - v_i^2\right) = Area = 25$$

$$v_f = \sqrt{\frac{2 \times 25 + v_i^2}{m}} = \sqrt{\frac{50 + 9}{m5}} = 4.36\ m/s$$

Q8.

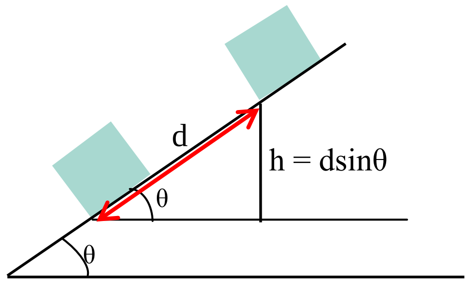
A block of mass *m* = 2.00 kg is placed against a spring on a frictionless incline making an angle θ = 30.0° with the horizontal, as shown in **Figure 6** (The block is not attached to the spring). The spring, with spring constant *k* = 1.96×10^3 N/m, is compressed 20.0 cm and then released. What is the maximum distance along the incline that the block will travel from the release point to its highest point on the incline? (Ignore air resistance)

Figure 6



A) 4.00 m
B) 5.81 m
C) 2.12 m
D) 3.00 m
E) 7.55 m

Ans:



$$U_{gi} + U_{si} = U_{gf} + U_{sf}$$

$$\frac{1}{2} k x^2 = mgh = mgd\sin\theta$$

$$d = \frac{kx^2}{2mg\sin\theta} = \frac{1960 \times (0.2)^2}{2 \times 2 \times 9.8 \times \sin 30} = 4.00$$

Q9.

A 4.0 kg block sliding on a frictionless surface breaks into two parts of equal masses. One part moves with a velocity of 3.0 m/s, due north, and the other part moves with a velocity of 5.0 m/s, 30° north of east. What was the initial sliding speed of the block? (Ignore air resistance)

A) 3.5 m/s
B) 2.2 m/s
C) 1.9 m/s
D) 4.2 m/s
E) 5.1 m/s

Ans:

$$m\vec{v}_i = \frac{m}{2}3\vec{j} + \frac{m}{2}5\sin30\vec{j} + \frac{m}{2}\,5\cos30\vec{i}$$

$$\vec{v}_i = \left(\frac{3}{2} + \frac{5}{2}sin30\right)\vec{j} + \frac{5}{2}cos30\,\vec{i} = 2.75\,\vec{j} + 2.17\,\vec{i}$$

$$|v_i| = \sqrt{(2.75)^2 + (2.17)^2} = 3.50\ m/s$$

Q10.

The angular speed of an automobile engine is increased at a constant rate from 1500 rev/min to 3000 rev/min in 12 s. How many revolutions does the engine make during this 12 s interval?

A) 4.5×10^2 rev
B) 2.9×10^2 rev
C) 3.8×10^2 rev
D) 5.5×10^2 rev
E) 6.1×10^2 rev

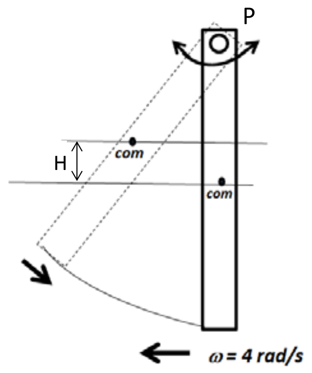
Ans:

$$\Delta\theta = \omega_{avg} \times \Delta t = \frac{\omega_i + \omega_f}{2} \times \Delta t = \frac{1500 + 3000}{2} \times \frac{12}{60} = 450 = 4.5 \times 10^2 \text{ rev}$$

Q11.

A thin uniform rod of length 0.75 m and mass 0.42 kg is suspended freely from one end. It is pulled to one side and then allowed to swing vertically like a pendulum. When its center of mass is at its lowest position, the rod has an angular speed of 4.0 rad/s. What is the maximum height H (as shown in **Figure 7**) above this lowest position to which the center of mass will rise? (Neglect friction and air resistance)

Figure 7



A) 0.15 m
B) 0.10 m
C) 0.35 m
D) 0.22 m
E) 0.05 m

Ans:

$$K_i + U_{gi} = K_f + U_{gf} \Rightarrow K_{rot-i} = U_{gf}$$

$$\frac{1}{2} I_p \omega^2 = MgH$$

$$\frac{1}{2} \frac{ML^2}{3} \omega^2 = MgH$$

$$H = \frac{L^2\omega^2}{6g} = \frac{(0.75)^2 \times 16}{6 \times 9.8} = 0.153\ m$$

Q12.

A 150 kg uniform solid sphere rolls along a horizontal floor so that the sphere's center of mass has a speed of 0.150 m/s. How much work must be done on the sphere to stop it?

A) – 2.36 J
B) – 5.11 J
C) – 3.15 J
D) – 4.54 J
E) – 1.05 J

Ans:

$$W_{app} = \Delta K = K_{rot-f} - K_{rot-i} = -K_{rot-i} = -\frac{1}{2} M v_{com}^2 - \frac{1}{2} I \omega^2$$

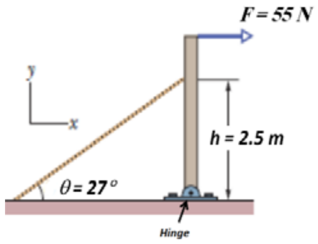
$$W_{app} = -\frac{1}{2} M v_{com}^2 - \frac{1}{2}\frac{2}{5} MR^2 \cdot \frac{v_{com}^2}{R^2} = -\frac{7}{10} M v_{com}^2 = -\frac{7}{10} \times 150 \times (0.15)^2$$

$$W_{app} = -2.36\ \text{J}$$

Q13.

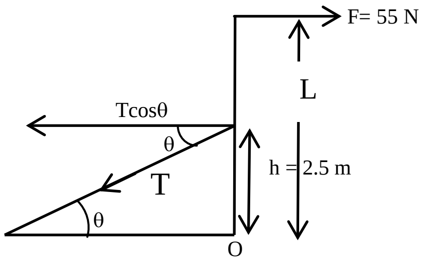
In **Figure 8** a uniform beam with a weight of 60 N and a length of 3.0 m is hinged at its lower end. A horizontal force $\vec{F}$ of magnitude 55 N acts at its upper end. The beam is held in static equilibrium vertically by a cable that makes an angle θ = 27° with the horizontal and is attached to the beam at height *h* = 2.5 m. What is the tension in the cable?

Figure 8



A) 74 N
B) 47 N
C) 32 N
D) 87 N
E) 91 N

Ans:

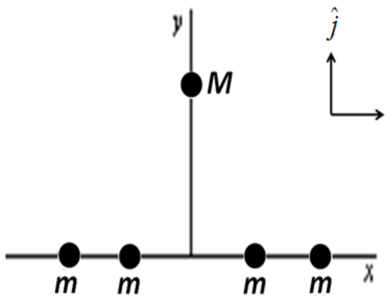


$$Taking\ moment\ about\ O \sum \tau_0 = T\cos\theta \times h - FL = 0$$

$$T = \frac{FL}{h\cos\theta} = \frac{55 \times 3}{2.5 \times \cos 27} = 74\ N$$

Q14.

Four particles, each with mass *m*, are arranged symmetrically about the origin on the *x* axis. A fifth particle, with mass *M*, is on the *y* axis as shown in **Figure 9**. The direction of the net gravitational force on *M* is along:



A) $-\hat{j}$
B) $-\hat{i}$
C) $+\hat{j}$
D) $+\hat{i}$
E) none of other answers

Ans:

$$\sum F_x = 0$$

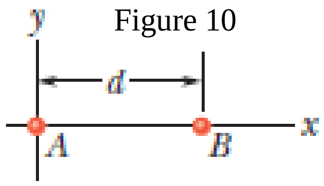
$$\sum F_y = -|F_{y-net}|\vec{j}$$

Q15.

In **Figure 10**, two point particles are fixed on an x axis separated by distance d = 3.50 m. Particle A, located at the origin, has mass m_A = 1.00 kg and particle B has mass m_B = 3.00 kg. A third particle C, of mass m_C = 75.0 kg is to be placed on the x axis and near particles A and B. At what x coordinate should C be placed so that the net gravitational force on particle A from particles B and C is zero?

A) –17.5 m
B) +21.1 m
C) +17.5 m
D) –21.1 m
E) –12.5 m

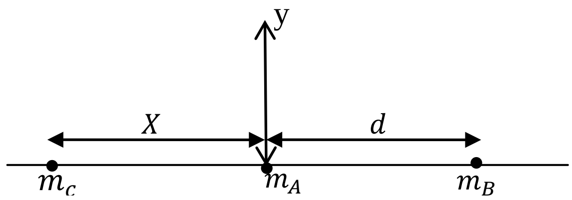
Figure 10



Ans:

$$\sum F_A = 0 \Rightarrow |F_{CA}| = |F_{AB}|$$

$$\frac{Gm_c m_A}{x^2} = \frac{Gm_A m_B}{d^2} \Rightarrow \frac{\sqrt{m_c}}{X} = \frac{\sqrt{m_B}}{d}$$



$$|X| = d\sqrt{\frac{m_c}{m_B}} = 3.5 \times \sqrt{\frac{75}{3}} = 3.5 \times 5 = 17.5 = -17.5\ m$$

Q16.

A uniform spherical planet of radius 500 km has a gravitational acceleration of 3.0 m/s^2 at its surface. With what speed will an object hit the planet's surface if it is released (v_{initial} = 0) from 1000 km height above the surface?

A) 1.4 km/s
B) 2.5 km/s
C) 3.2 km/s
D) 1.9 km/s
E) 2.0 km/s

Ans:

$$E_i = E_f$$

$$\frac{1}{2}mv_i^2 - \frac{GmM}{R_i} = \frac{1}{2}mv_f^2 - \frac{GmM}{R_o} \quad (v_i = 0)$$

$$v_f^2 = 2GM\left(\frac{1}{R_o} - \frac{1}{R_i}\right) = 2GM\left(\frac{R_i - R_o}{R_i R_o}\right);$$

$$v_f = \sqrt{\frac{2GM}{R_i R_o}(R_i - R_o)} = \sqrt{\frac{2 \times a_g R_o^2}{R_i R_o}(R_i - R_o)}$$

$$R_i = 1000 + 500 = 1500\ km$$

$$R_o = 500\ km$$

$$\frac{Gm}{R_o^2} = a_g \Rightarrow GM = a_g R_o^2$$

$$a_g = 3 m/s^2 = 3 \times 10^{-3}\ km/s^2$$

$$v_f = \sqrt{\frac{2 \times a_g \times -R_o}{R_i}(R_i - R_o)} = \sqrt{\frac{2 \times 3 \times 10^{-3} \times 500}{1500}(1500 - 500)} = \sqrt{2} = 1.4\ km/s$$

Q17.

A 20.0 kg satellite moves on a circular orbit around a planet of mass M = 4.06×10^{24} kg with a period of 2.40 h. What is the radius of the orbit of the satellite?

A) 8.00×10^{6} m
B) 4.32×10^{6} m
C) 1.11×10^{6} m
D) 5.57×10^{6} m
E) 9.34×10^{6} m

Ans:

$$T^2 = \frac{4\pi^2}{GM} \times r^3 \Rightarrow r = \left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}}$$

$$r = \left[\frac{6.67 \times 10^{-11} \times 4.06 \times 10^{24} \times (2.4 \times 3600)^2}{4\pi^2}\right]^{1/3} = 8.00 \times 10^6 \text{ m}$$

Q18.

A U-tube has dissimilar arms. The diameter of one arm is twice the diameter of the other arm. It contains an incompressible fluid and is fitted with a sliding piston in each arm, with each piston in contact with the fluid. When the piston in the narrow arm is pushed down a distance d, the piston in the wide arm rises a distance:

A) d/4
B) 2d
C) d/2
D) d
E) 4d

Ans:

$$V_{\text{narrrow}} = V_{\text{wider}} \Rightarrow \pi r^2 \times d = \pi (2r)^2 \times d'$$

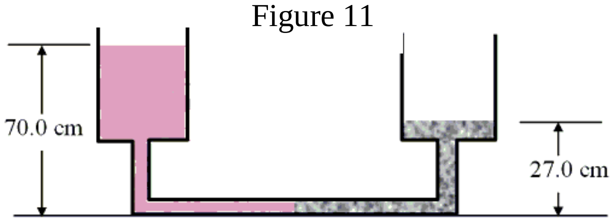
$$d = 4d' \Rightarrow d' = \frac{d}{4}$$

Q19.

A column of oil of height 70.0 cm supports a column of an unknown liquid as shown in **Figure 11** (not drawn to scale). Assume that both liquids are at rest and that the density of the oil is 840 kg/m^3. Determine the density of the unknown liquid.

A) 2.18×10^3 kg/m^3
B) 3.62×10^3 kg/m^3
C) 2.96×10^3 kg/m^3
D) 1.23×10^3 kg/m^3
E) 1.09×10^3 kg/m^3

Figure 11



Ans:

$$P_A + \rho_{oil} \times g \times h_{oil} = P_A + \rho_x \times g \times h_x$$

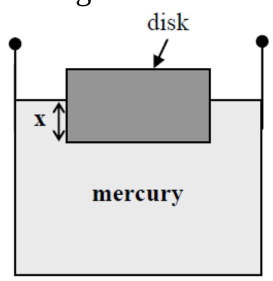
$$\rho_x = \frac{\rho_{oil} \times h_{oil}}{h_x} = 840 \times \frac{0.7}{0.27} = 2.18 \times 10^3 kg/m^3$$

Q20.

A disk made of lead (diameter = 9.0 cm, height = 7.0 cm, density = 11.3×10^3 kg/m^3) floats in a container, which is filled with mercury (density = 13.6×10^3 kg/m^3) as shown in **Figure 12**. What is the depth **x** by which the disk sinks in mercury?

A) 5.8 cm
B) 4.3 cm
C) 1.2 cm
D) 6.8 cm
E) 3.7 cm

Figure 12



Ans:

$$m_{Pb} \times g = V_{submerged} + \rho_{Hgx} \times g$$

$$\pi r^2 \times h \times \rho_{pb} \times g = \pi r^2 \times x \times \rho_{Hg} \times g$$

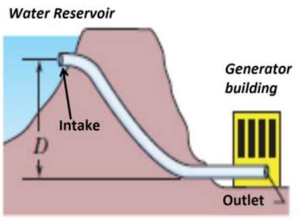
$$x = h \cdot \frac{\rho_{pb}}{\rho_{Hg}} = 0.07 \times \frac{11.3 \times 10^3}{13.6 \times 10^3} = 0.058\, m = 5.8 \text{ cm}$$

Q21.

The intake of the pipe shown in **Figure 13** has a cross-sectional area of 0.74 m^2 and the water from the reservoir flows into the pipe at a speed of 0.40 m/s. At the outlet, a distance D = 180 m below the intake, the cross-sectional area of the pipe reduces to 0.03 m^2 and the water flows out of the pipe into a generator building. What is the pressure difference between the outlet and intake of the pipe?

A) 1.7×10^6 Pa
B) 2.9×10^6 Pa
C) 4.3×10^6 Pa
D) 1.1×10^6 Pa
E) 3.2×10^6 Pa

Figure 13



Ans:

$$P_{in} + \rho g D + \frac{1}{2}\rho v_{in}^2 = P_{out} + \frac{1}{2}\rho v_{out}^2$$

$$v_{in}A_{in} = v_{out}\cdot A_{out} \Rightarrow v_{out} = v_{in}\cdot\frac{A_{in}}{A_{out}} = 0.4\times\frac{0.74}{0.03} = 9.87\ m/s$$

$$P_{out} - P_{in} = \rho g D + \frac{1}{2}\rho(v_{in}^2 - v_{out}^2)$$

$$P_{out} - P_{in} = 10^3\times9.8\times180 + \frac{1}{2}\times10^3\times((0.4)^2-(9.87)^2) = 1.71\times10^6\ \text{Pa}$$

Q22.

Which of the following relationships between the acceleration a and the displacement x of a particle represents a simple harmonic motion (SHM)? (I) $a = 0.1x$, (II) $a = 100x^2$, (III) $a = -10x$, (IV) $a = -5x^2$

A) III
B) I
C) IV
D) II
E) None of the other answers

Ans:

A

Q23.

The equation $x = 6.0 \cos([\pi \text{ rad/s}]\, t + \pi/3 \text{ rad})$, where x is in meters and t is in seconds, describes the simple harmonic motion of a body. What is the speed of the body at $\boldsymbol{t}$ **=1.0 s**?

A) 16 m/s
B) 11 m/s
C) 14 m/s
D) 22 m/s
E) 27 m/s

Ans:

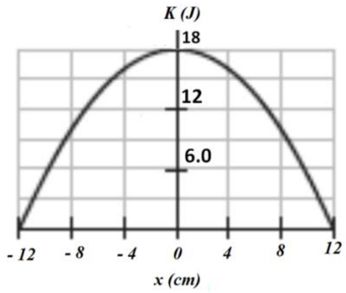
$$v = \frac{dx}{dt} = -6\pi sin\left[\pi t + \frac{\pi}{3}\right]$$

$$v(t = 1s) = -6\pi sin\left[\pi + \frac{\pi}{3}\right] = -6\pi sin\left(\frac{4\pi}{3}\right) = 16.3 \text{ m/s}$$

Q24.

Figure 14 shows the kinetic energy K of a simple harmonic oscillator (horizontal mass-spring system) versus its position x. What is the spring constant of the harmonic oscillator?

Figure 14



A) 2.5×10^3 N/m
B) 1.3×10^3 N/m
C) 1.1×10^3 N/m
D) 3.7×10^3 N/m
E) 3.5×10^3 N/m

Ans:

$$E = K_{max} = U_{max} = \frac{1}{2}kx_{max}^2$$

$$k = \frac{2K_{max}}{x_{max}^2} = \frac{2\times18}{(0.12)^2} = 2500N/m = 2.5\times10^3\ N/m$$

Q25.

A thin uniform rod of 0.50 kg mass swings vertically in simple harmonic motion about an axis that passes through one end of the rod and is perpendicular to the plane of the swing. The rod swings with a period of 1.1 s. What is the length of the rod?

A) 0.45 m
B) 0.02 m
C) 0.33 m
D) 1.1 m
E) 2.4 m

Ans:

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{m L^2/3}{mg\, L/2}} = 2\pi\sqrt{\frac{2}{3}\frac{L}{g}} = \sqrt{\frac{8\pi^2 L}{3g}}$$

$$L = \frac{3g}{8\pi^2}T^2 = \frac{3 \times 9.8 \times (1.1)^2}{8\pi^2} = 0.45\text{ m}$$